THE EFFECT OF THE WAR RISK: A COMPARISON OF THE CONSEQUENCES OF THE TWO IRAQ WARS ON SOME FINANCIAL VARIABLES

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Introduction

This paper empirically measures the effect of war on some Italian and US financial variables using the heteroskedasticity based estimation technique proposed by Rigobon and Sack (2003). This work refers to the days of the two wars with Iraq, and considers the effects of these wars in Italy and the effect of the Iraq war of 1990 in US as well. This study has been motivated by "The effect of war risk in US" where Rigobon and Sack (2005) study the effects of the Iraq war of 2003 analysing its effects in US.

The aims of this work are twofold. The first is to verify whether the two Iraq wars has had similar effects. We analyse whether the war has had different results in the two periods and we test the robustness of Rigobon and Sack's methodology as well. Second, we measure the effect of war on other variables beyond those considered by Rigobon and Sack.

This approach is innovative because of the difficulty of identifying the effect of war, which is a variable that is not measurable, on some variables. If the risk of war was a variable observable and measurable, then we could have simply used an OLS to estimate its effects on the economy.

The advantage of the heteroskedasticity-based approach we are going to apply is that it allows to identify the impact of war risk without having to quantify or even sign the risk itself. Implementing this estimator only requires to identify two sets of days: one in which the variance of war related news was high and one in which the variance of war related news was low. These days have been identified in accordance to the events and news which have been considered as having a significant effect for the war. Rigobon and Sack' estimation technique indicates that determining this set of days is sufficient to capture the effects of the level of war risk factor on various asset prices.

This feature of the methodology is appealing because it is much easier to determine when war related news took place, than to quantify the news itself.

In particular, identification of the parameters is obtained by the shift in the variance of the variables on these days; this is sufficient to capture the effect of war on the variables themselves.

When we refer to the Iraq war of 2003, the findings of this study suggest that the risk of war has a significant role in explaining the behaviour of Italian financial variables. The financial variables considered are the equity price index, the average yield of a 10 and 5 years government bond, the 30 years, 10 years and 2 years government bond price index, the gold price, the nominal euro effective exchange rate and the interbank rate at one month.

As the results indicate, increases in the risk of war produce significant effects. The increase in the risk of war caused a significant increase in the government bond price index at all maturities, a significant decline in the equity price index and in the average yield of the bonds, and finally a depreciation of the dollar.

These results show the importance that an event such as the war in Iraq has had in explaining the movement and behaviour of the financial variables of that period. The results we find are in line with those found by Rigobon and Sack (2005) for US. This finding shows the equal response of the behaviour of different countries to the war; it can also be interpreted as a proof of robustness of Rigobon and Sack's findings.

When we refer to the Iraq war of 1990, instead, the results suggest that the risk of war did not affect the behaviour of the financial variables in both Italy and US. The financial variables considered, because of the inferior number of data available, are only the equity price index, the 2 years government bond price index, the exchange rate and the interbank rate at one month for Italy; the equity price index, the 10 years and 2 years government bond price index, the exchange rate and the interbank rate at one month for US.

Different from previous findings, here none of these variables reacted in a significant way to the war in Iraq.

The structure of the war proceeds as follows. We first describe the methodology applied, underlying its main advantages. We then apply it to my data and finally we present the results followed by the conclusion.

The Heteroskedasticity Based Estimation

Before showing the results, we briefly summarise the most important features belonging to the heteroskedasticity-based technique. The heteroskedasticity based approach, which has been

suggested by Rigobon and Sack and applied by the authors in many analysis (2003, 2004, 2005) relies on the heteroskedasticity in the data in order to identify the parameters.

This identification approach assumes that the shift in the second moment of the financial variables during particular days is sufficient to explain the behaviour of these variables on those days. In particular, the methodology relies on the change in the covariance of the financial variables (we consider two of them at a time) on the days of war news, as it is plausible that during these days these variables are more volatile due to their responses to the war news.

We suppose to be able to identify a period of time in which the variance of these variables was higher than at other times. We only need to identify two sub samples, one in which the risk of war is elevated, "war days", and one in which the risk of war is low, "non war days". This assumption implies that the "importance" of the risk of war increases in the first sub sample; in this set of days the variance of this shock is elevated.

For the set of "war days" we choose the same period studied by Rigobon and Sack (2005) for the Iraq war of 2003; we select an other set of days for the Iraq war of 1990. These periods include all days in which events regarding the war in Iraq appeared on the newspaper. The "non war" days is the set of days immediately surrounding (preceding or following) the days included in the "war days" sample.

Thus, we consider two sets of days: the "war" days and the "non war" days and we assume that only the variance of the war risk factor changes during the "war" days. Of course, other factors can be present in the analysis, but with the same intensity throughout the all period considered.

We could have tested the effect of the war on the change in the financial variable resolving the problem due to the fact that the variable is not measurable with an OLS of the change of the financial variables on a dummy variable present on those days. But this methodology implies some problems. First, there are other factors which possibly influence continually and in a significant way the financial variables. Then, as shown in Rigobon and Sack, the average change of the financial variables does not differ throughout the two periods. One reason of this is that the "war days" selects some days associated with increases in the risk of war, other days associated with decreases of this risk and lastly some days for which it is difficult to understand the sign of the news.

The approach we here apply is based on the variance of the variables which, because of the greater intensity of war related news on "war days", has been proven to increase sharply on those days. This behavior is explained by the greater intensity of war related news on those days.

Thus, we assume that the changes in the two financial variables that we have considered can be explained by the following linear system

(1)

$$A\begin{bmatrix}\Delta x_1\\ \Delta x_2\end{bmatrix} = B\begin{bmatrix}z_1\\ z_2\\ z_3\\ \dots\end{bmatrix} + \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}$$

Considering two financial variables at a time, we indicate with Δx_1 the change in the first financial variable and with Δx_2 the change in the second financial variable. These are determined by a set of common factors $z = [z_1, z_2, \dots]^1$ which include news regarding the risk of war, and other macroeconomic events and a set of idiosyncratic shocks $\eta = [\eta_1, \eta_2]^1$. We call z_1 the effect of the risk of war.

The reduced form representation of system (1) is:

(2)

$$\begin{bmatrix}\Delta x_1\\ \Delta x_2\end{bmatrix} = D\begin{bmatrix}z_1\\ z_2\\ z_3\\ \dots\end{bmatrix} + \begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix},$$

where $D = A^{-1}B$ represents the effect of all of the factors on the financial variables and $\mu = [\mu_1, \mu_2]^1 = [A^{-1}\eta]$.

The elements of the matrix D can be written as:

(3)

$$D = \begin{bmatrix}1 & d_{12}\ d_{13} & ..\\ d_{21} & d_{22}\ d_{23} & ..\end{bmatrix}$$

where, indicating with z_1 the risk of war, the first column represents the effect of war risk on the two financial variables. In particular, d_{11} is the effect of war on the first financial variable. Because z_1 is unobservable, we can identify the model only up to a normalization. For this reason, we impose the impact on the first variable to be one.

d_{21} is what we are looking for; it represents the effect of the risk of war on the second financial variable.

As we have already reminded, there are two difficulties, which arise in estimating a model like that in equation (2). First, not all the variables are observable and measurable. If the

common factors were all observable then equation (2) could simply be estimated using an OLS regression. Second, many of these unobservable factors can influence the behaviour of the variables in a manner that makes it difficult to recover the impact of one of them on the variables themselves.

The approach used here allows to solve these problems only assuming the existence of two sub sample; in the "war days" sub period it is assumed that only the variance of the war risk factor shifts.

The approach estimates the risk of war effect through the computation of the variance covariance matrices of the two sets of days.

We compute this variance covariance matrix for the set of "war days", denoted as Ω_H, and for the set of "non war" days, denoted as Ω_L, assuming that the change in the variance of the variables between the two sets of days is due to z_1:

$$\Omega = \begin{bmatrix} \sigma_{z1}^2 & \sigma_{z1}^2 \cdot d_{21} \\ d_{21} & d_{21}^2 \cdot \sigma_{z1}^2 \end{bmatrix}.$$

As can be seen from equation (4), the change in the variance covariance matrix among these sub samples, $\Delta\Omega = \Omega_H - \Omega_L$ which is assumed to be due to the war risk factor, allows the identification:

(4)

$$\Delta\Omega = \Delta\sigma^2(z_1) \cdot \begin{bmatrix} 1 & d_{21} \\ d_{21} & d_{21}^2 \end{bmatrix}$$

where $\Delta\Omega = \Delta\sigma^2(z_1)$ is the shift in the variance of the war risk factor.

From equation (4) we can derive several estimates of the parameter d_{21}, as follows:

(5)

$$\hat{d} = \Delta\Omega_{22} / \Delta\Omega_{21}$$

(6)

$$\hat{d} = \Delta\Omega_{21} / \Delta\Omega_{11}$$

where

$$\Delta\Omega_{11} = \Delta\sigma^2(z_1);\ \Delta\Omega_{21} = \Delta\Omega_{12} = \Delta\sigma^2(z_1)d_{21};\ \ \Delta\Omega_{212} = \Delta\sigma^2(z_1)d_{21}{}^2.$$

Two important assumptions have been made in the analysis. The first is that the variance of the war risk factor increases in the "war days". The second is that the variance of other factors which might influence the financial variables remains unchanged in the two sub samples. A rejection that the coefficient estimates d are the same could indicate that one of these

assumptions is violated. There could be other shocks which impact on the financial variables in the two sets of days or there could be non-linearities which are not captured by the specification of the model.

As shown in Rigobon and Sack (2002), an interesting feature of the methodology just described is that it can be implemented by an instrumental variable technique. Maintaining the basic assumption of the methodology described, we here test the effects of the war with the help of instrumental variables.

To arrive at an instrumental variable interpretation of the estimator we have to define the instrument to be the change in the first financial variable, Δx_1, on all war news days, and the negative of its change, - Δx_1, on the " non war news days":

(7)

$$\omega_1 = \{\Delta x_{1,t}, \forall t \in H\} \cup \{-\Delta x_{1,t}, \forall t \in L\}$$

where H and L denote the set of war risk days and other days respectively. Rigobon and Sack (2002) show that the estimates of the impact of the war risk factor on the financial variables of equation (6) can be derived regressing the change in the first financial variable on the change in the second financial variable using both sets of days and the instrumental variable approach. The IV estimator is:

(8)

$$\hat{d} = \left(\omega_1^{'} \cdot \Delta x_1\right)^{-1} \cdot \left(\omega_1^{'} \cdot \Delta x_2\right),$$

which can be written as:

(9)

$$\hat{d} = \frac{\{\Delta x_{1_H}, -\Delta x_{1_L}\}' \{\Delta x_{2_H}, \Delta x_{2_L}\}}{\{\Delta x_{1_H}, -\Delta x_{1_L}\}' \{\Delta x_{1_H}, \Delta x_{1_L}\}} = \frac{Cov_H(\Delta x_1, \Delta x_2) - Cov_L(\Delta x_1, \Delta x_2)}{Var_H(\Delta x_1) - Var_L(\Delta x_1)}$$

where the subscripts H and L indicate the set of days over which the variance and the covariances are taken. The coefficient (9) is identical to the estimator (6).

In the same way we can define an alternative instrument only using the second financial variable:

(10)

$$\omega_2 = \{\Delta x_{2,t}, \forall t \in H\} \cup \{-\Delta x_{2,t}, \forall t \in L\}$$

With this instrument, the IV estimator becomes

(11)

$$\hat{d} = \frac{\left\{\Delta x_{2_H}, -\Delta x_{2_L}\right\}'\left\{\Delta x_{2_H}, \Delta x_{2_L}\right\}}{\left\{\Delta x_{2_H}, -\Delta x_{2_L}\right\}'\left\{\Delta x_{1H}, \Delta x_{1_L}\right\}} = \frac{Var_H(\Delta x_2) - Var_L(\Delta x_2)}{Cov_H(\Delta x_1, \Delta x_2) - Cov_L(\Delta x_1, \Delta x_2)}$$

which is identical to the estimator (5) above.

Rigobon and Sack (2002) have shown that both ω_1 and ω_2 are valid instrument for estimating d under the assumptions made- that the parameters are stable and that the war risk factor is heteroskedastic. Their proof is reported in the appendix.

Defining ω_1 and ω_2 as valid instruments for the variables, we find the effect of the war risk factor regressing the change in the first financial variable on the change in the second financial variable.

Application of the methodology: data and results

To implement the methodology described, as said above, we first have to identify two sets: one in which the variance of war risk was elevated and one in which the variance of the war risk factor was low. We have used the 17 days selected by Rigobon and Sack for the Iraq war of 2003. We have constructed an other series of days for the Iraq war of 1990. These are the days on which war related events appeared to be the primary cause of the behaviour and movement of the financial variables. In this set of days is thus plausible to assume that only the variance of the war risk factors shifts. The list of these days is replicated in Table 1 and Table 2.

Using these two sub samples we apply the instrumental variable described above to some financial variables. The countries which have been analysed are Italy for the war of 2003 and also US for the war of 1990. In particular, as is evident in Table 3, the financial variables considered are the equity price index, the average yield of a 10 and 5 years government bond, the 30 years, 10 years and 2 years government bond price index, the gold price, the exchange rate and the interbank rate at one month for Italy. The variables studied for US are the equity price index, the 10 years and 2 years government bond price index, the effective exchange rate, and the interest rate at one month. The data are taken from Bloomberg and Datastream.

As described above, we make the analysis considering two financial variables at a time, and we study the impact of a change in z1 by –0.25. Thus, the reported coefficients of Table 3 represent movements induced by an increase in the war risk that is large enough to cause a 0.25 basis point drop in the first financial variable that we consider in the analysis.

Looking at the Iraq war of 2003, which is the same episode studied by Rigobon and Sack, the primary finding of this paper is that, in accordance to what has been studied for US, the risk

of war has a significant effect on many of Italian financial variables that we have included in the study.

The results are reported in Table 3 which shows the coefficients obtained under the two instruments determined above and in brackets their significance levels.

Having a general look at the findings we can at first observe that the coefficients obtained using both the sets of instruments are typically close to one another. This makes the structure that we are assuming in the analysis reliable. Thus we can think that the variance of the financial variables that we have analysed in the set of days we have considered, can be explained by the war shocks.

In Italy the increase in the risk of war of the magnitude that we have assumed above has shown a significant effect on the financial variables. While the stock prices have had a significant decline, there has been a significant increase in the government bond price index at all maturities. This result indicates that the risk of war has led people to fear the possibility of a recession, with their consequent movement of preference, from risky assets, (and the following decline of their prices), to less risky and safer assets (with the following increase in their prices). The risk of war has had a significant impact on the average yields of bonds, as well. An increase in the risk of war caused a decline in the average yields of bonds, as their prices increased.

As in Rigobon and Sack (2005), surprisingly, no effect has been found on gold price and on CPI. This is in contrast with the hypothesis made according to which people prefer safer to riskier assets. This is a strange result; we would have expected an increase in the price of gold as people during war and conflicts are expected to prefer safer assets thus increasing their prices. We would have also expected an increase in the CPI as the experience teaches us that after a war the economy experiences a period of high inflation. The reason of our results can be probably explained by the fact that we are here studying the immediate effects of the war, while the mentioned effects are likely to happen at a longer term.

Finally, the risk of war shows a significant effect on the exchange rate: consistent with Rigobon and Sack's (2005) findings the risk of war caused a depreciation of the dollar.

Contrary to these results, looking at the test on the Iraq war of 1990, we do not find a significant movement on the variables for both Italy and US. As table 3 makes clear, while the sign of the impact of the war on the variables is equal to that one described for the war of 2003, the risk of war of 1990 had a lower impact on people confidence. After the Iraq war of 2003 in fact people possibly feared that the conflict would have lasted longer. This made them shifting their preferences to safer assets with the consequent effect on the behaviour of the financial variables. The Iraq war of 2003 presumably worsens the economic situation of a country which was already facing a period of uncertainty. This is probably the reason of the strong response of

the financial variables to the war. The Iraq war of 1990, instead, was probably believed as a temporary situation and this is probably the explanation of the fact that people did not change their preferences and the financial variables did not move as a consequence of the conflict.

Conclusions

This paper has demonstrated that the risk of war has had significant effects on some Italian financial variables when we consider the Iraq war of 2003; no result has been found in both Italy and US, when we refer to the Iraq war of 1990.

This analysis has been conducted through the heteroskedasticity based estimation approach. The basis of this analysis has been to divide the period of study into two set of days, one in which the heteroskedasticity of the war news has been elevated and one in which it has been low. The set of "war days" includes days in which there has been important news connected with the war. The "non war days" are days proceeding or postponing the "war days".

When we refer to the Iraq war of 2003, the results suggest that the risk of war has a significant role in explaining the behaviour of the Italian financial variables.

The increase in the risk of war caused a significant increase in the government bond price index at all maturities, a significant decline in the equity price index and in the average yield of the bonds, and finally a depreciation of the dollar. For contrast there does not seem to be a significant effect of the war risk on the gold price.

Contrary to these results, looking at the test on the Iraq war of 1990, in both Italy and US, we do not find any significant movement on the variables. This suggests that the war was probably understood by the countries as a temporary situation; this feeling was responsible of the fact that none of the analysed variables reacted in a significant way to the war.

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Table 1
Dates of High Variance of War Risk - Iraq war of 2003

Date	Event	War Risk
1/9/2003	U.N inspectors report finding no chemical weapons Reports that N. Korea will abandon nuclear arms program if U.S. reaffirms non hostility agreement	Decreased Decreased
1/10/2003	N. Korea announces withdrawal from unclear non-proliferation treaty	Increased
1/16/2003	Reports that Saddam Hussein might consider exile U.N. weapons inspectors find empty chemical warheads	Decreased Increased
1/17/2003	Saddam Hussein gives speech stating that Iraq is ready for war	Increased
1/27/2003	Blix report: "Iraq appears not to have come to a genuine acceptance of the disarmament"	Increased
1/29/1003	President Bush gives State of Union Address Secretary Powell says U.S. would assist Saddam Hussein if he sought exile	Unclear Decreased
1/30/2003	President Bush comments on continued lack of Iraq cooperation	Increased
2/5/2003	Secretary Powell makes U.N representation in effort to build a broad coalition	Unclear
2/10/2003	Reports that Iraq will unconditionally allow surveillance flights	Decreased
2/12/2003	Secretary Powell says impasse has reached "moment of truth" U.S. intelligence says N. Korea can reach U.S. with nuclear missle	Increased Increased
2/13/2003	Rumours that President Bush set deadline to attack without resolution	Increased
2/14/2003	Blix report interpreted as reducing chance of immediate war	Decreased
3/5/2003	Secreatary Powell makes tough comments on Iraq	Increased
3/7/2003	Reports that Bin Laden close to being captured	Decreased
3/10/2003	Turkey rejects U.S. use of military bases	Unclear
3/13/2003	CNN reports that Iraq might surrender before conflict begins	Decreased
3/17/2003	President Bush expected to announce an ultimatum with a short deadline for war	Increased

Table 2
Dates of High Variance of War Risk - Iraq war of 1990

Date	Event	War Risk
2/8/1990	The war in Iraq breaks out. Saddam Hussein' armies invade the emirate	Increased
6/8/1990	UN Security Council decides an embargo against Iraq and the occupied territories	Unclear
8/8/1990	Iraq annexes Kuwait	Increased
20/8/1990	Iraq announces that all citizens belonging to the "aggressive" countries will be held in strategic locations	Increased
27/8/1990	UN Security Council approves to use violence in order to respect the embargo	Increased
28/8/1990	President Hussein decrees Kuwait an Iraqui province. Moreover Husseins authorizes all foreign women and children to leave Iraq.	Unclear
25/9/1990	UN Security Council with the resolution 670 decrees embargo against Iraq	Increased
6/12/1990	President Hussein announces the liberation of all western hostages.	Decreased
17/1/1991	The alarm is sounding in Baghdad and the antiaircraft artillery opens fire in the capital	Increased
23/1/1991	Powel announces that in one week 12000 raids take place successfully over enemy targets, hitting the 95% of Iraqui radars	Decreased
25/2/1991	Saddam gives order to his armies to withdraw to the same posts of the 1990s	Decreased
26/2/1991	The Iraqui army withdraws	Decreased

Table 3
Estimated Impact of Increase in the risk of war
a) Italy

Variable	**Eqn (6) IV**	**Eqn (7) IV**	**Eqn (6) IV**	**Eqn (7) IV**
The Iraq war of 2003			**The Iraq war of 1990**	
5-yearYield Government Bond	-1.0087 (46.2888)*	-1.0072 (47.6224)*		
Equity Price Index	-0.2224 (4.3267)*	-0.0615 (0.6912)*	0.1648 (0.5598)	-0.0374 (-0.1562)
30-years Government Bond Price Index	0.1148 (-7.2382)*	0.1139 (-6.2306)*		
10-years Government Bond Price Index	0.0615 (-8.9307)*	0.0567 (-6.0152)*		
2-years Government Bond Price Index	0.0147 (-5.9973)*	0.0154 (-4.8494)*	0.0916 (-0.0381)	0.0013 (-0.2953)
Gold Price	-0.0007 (0.5752)	0.1188 (-0.2849)		
Effective Exchange rate	0.0376 (-2.3905)*	0.0699 (-1.5755)	0.2966 (-0.0377)	0.0322 (-0.4838)
Interbank rate	0.0528 (-1.0655)	0.6637 (-1.0763)		

-T statistics in brackets
- * means statistically significant
- The first variable is the interest rate at one month. Thus the reported coefficients represent movements induced by an increase in the war risk that is large enough to cause a 0.25 basis point drop in the interest rate

Estimated Impact of Increase in the risk of war
b) US

Variable	**Eqn (6)** **IV**	**Eqn (7)** **IV**
10 years Government Bond Price Index	-0.0002 (0.0477)	0.7478 (-0.0502)
2 years Government Bond Price Index	0.0029 (-0.3689)	0.0002 (-0.318)
Effective Exchange Rate	-0.0014 (0.2350)	0.1087 (-0.1715)
Equity price index	-0.0031 (-0.1033)	0.2087 (-0.1715)

- T statistics in brackets
- The first variable is the interest rate at one month. Thus the reported coefficients represent movements induced by an increase in the war risk that is large enough to cause a 0.25 basis point drop in the interest rate

Appendix

We investigate the validity of the instruments:

a) an instrument, in order to be valid, has to be correlated with the regressors, but uncorrelated with the residuals.

b) an instrument is valid if the parameters of the model are stable.

c) in order for the instrument to be valid, the assumptions on the heteorskedasticity of the war shocks has to be satisfied.

From the validity of the instruments, the asymptotic properties of the estimator derive.

a) If the variance of η_t is constant between the war days *(H)* and the non-war days (L) then ω_1 and ω_2 are all valid instruments to estimate d.

The proof of this proposition follows:

A valid instrument needs to be correlated with the explanatory variables from the regression but uncorrelated with the residuals. The reduced form of the above equations is represented by equation:

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = D \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \dots \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix},$$

We want to estimate the war effect z_1 on the two financial variables x_1 and x_2.

Under the assumptions that only the war shocks increases throughout the subsamples:

$$\sigma_{z_1}^H = \sigma_{z_1}^L + \Delta\sigma_{z_1}$$

$$\sigma_\eta^H = \sigma_\eta^L$$

each instrument is correlated with the explanatory variables:

$$p\lim \frac{1}{T}\omega_1^{'}\Delta x_1 = \frac{1}{T}\sum_{t\in H}\Delta x_1^2 - \frac{1}{T}\sum_{t\in L}\Delta x_1^2$$

$$\rightarrow Var(\Delta x_1 \mid H) - Var(\Delta x_1 \mid L)$$

$$= \Delta\sigma^2(z_1) \neq 0$$

$$p\lim \frac{1}{T}\omega_2^{'}\Delta x_1 = \frac{1}{T}\sum_{t\in H}\Delta x_2\Delta x_1 - \frac{1}{T}\sum_{t\in L}\Delta x_2\Delta x_1$$

$$\rightarrow Cov(\Delta x_2, \Delta x_1 \mid H) - Cov(\Delta x_2, \Delta x_1 \mid L)$$
$$= d_{21} \Delta\sigma^2(z_1) \neq 0$$

and each instrument is uncorrelated with the residuals. With this aim, we show that each instrument is uncorrelated with each of the structural shocks, η:

$$p\lim \frac{1}{T}\omega_1^{'}\eta = \frac{1}{T}\sum_{t\in H}\Delta x_1 \eta_t - \frac{1}{T}\sum_{t\in L}\Delta x_1 \eta_t \rightarrow \sigma_\eta^H - \sigma_\eta^L = 0$$

Similar equations are obtained for the other instrument:

$$p\lim \frac{1}{T}\omega_2^{'}\eta = \frac{1}{T}\sum_{t\in H}\Delta x_2 \eta_t - \frac{1}{T}\sum_{t\in L}\Delta x_2 \eta_t \rightarrow \sigma_\eta^H - \sigma_\eta^L = 0$$

b) If the parameters of the model are unstable then the instruments are not valid. This happens when there are non linearities in the model, or when the parameters shift in policy dates. One advantage of the model is that if the model is non-linear or if the parameter shifts, then the overidentifying assumptions is rejected.

c) in order for the instrument to be valid, the assumptions on the heteroskedasticity of the stock market shocks and of the common shocks have to be satisfied.

The proof of sentence b) and c) derives from proposition a).