# LECTURES ON MEASUREMENT SCIENCE*

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# 1. Models of the measurement process

**Abstract -** *The epistemic requirement that measurement be an objective and intersubjective evaluation is empirically fulfilled by adopting measuring systems that include selective and repeatable sensors and traceable standards; any measurement is then performed as a (direct or indirect) comparison to a chosen standard.*

*From the instrument output the measurement result has to be inferred by means of a process based on the information gathered from the instrument calibration and a measurement system model. Such a process is only plausible in its results, that must be expressed specifying both a measurand value and its estimated uncertainty.*

## 1.1. Measurement as a comparison model

Measurement is an operation of data acquisition and presentation, aimed at expressing in symbolic form the information empirically obtained on a system about a quantity, the *measurand* (we accept the common ambiguities of calling "measurand" both the system under measurement and the measured quantity, and the latter in both its general and specific forms, e.g., length and length of a given object in a given time). Peculiar to measurement is the requirement of *being objective and intersubjective*, where objectivity implies that measurement results convey information only related to the system under measurement and not its environment, and intersubjectivity requires that measurement results convey the same information to different subjects. As such, these properties appear an ideal target, justifying the efforts to constantly enhance measurement devices and procedures.

To achieve an acceptable degree of objectivity and intersubjectivity measuring systems are adopted, including selective and repeatable sensors and traceable standards. Indeed:

- although human beings are able to directly sense a fair amount of quantities and are well trained to express in linguistic form their perception (e.g., "it is rather cold", "this is heavier than that"), their statements are affected by subjectivity, i.e., they report information on both the sensed system and the perceiver state; to avoid the influence of the latter, and thus to enhance the objectivity of the operation, the measurand is transduced by a sensing system whose output ideally depends only on the measurand and is unaffected by influence quantities and internal imperfections;
- while related to the measurand, the quantity provided by sensors still depends on their specific behavior; as a consequence, distinct sensors, even if perfectly repeatable, produce different outputs from the same input; furthermore, in many cases the sensor output quantity, appropriate for signal conditioning and for driving presentation devices, is not dimensionally homogeneous to the measurand. The sensor output must be then dealt with *as an instrument reading, not a measurand value*. To make the information obtained by the measurement intersubjective a common reference must be adopted, so that measurand values are expressed in comparison to such a standard. Critical is therefore the possibility

to trace the readings to the agreed standard, a condition operatively ensured by instrument calibration.

The requirement of empirical comparison to traceable standards is so fundamental that can be assumed as distinctive of measurement; if generic scale-preserving evaluations can be formalized as homomorphisms from empirical to symbolic relational systems:

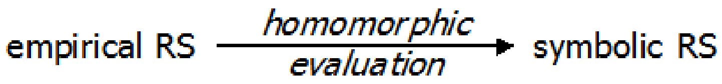


Figure 1 – A generic scale-preserving evaluation.

in the case of measurement such mappings are not direct but mediated by the comparison to standards:

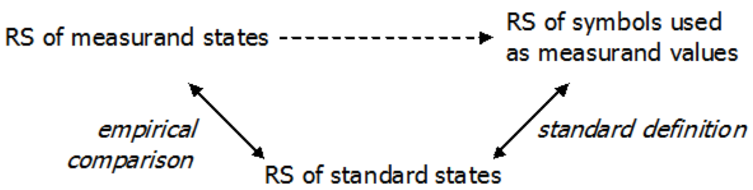


Figure 2 – Measurement as a scale-preserving evaluation obtained by the comparison to a standard.

Finally, when primary standards are not directly available:

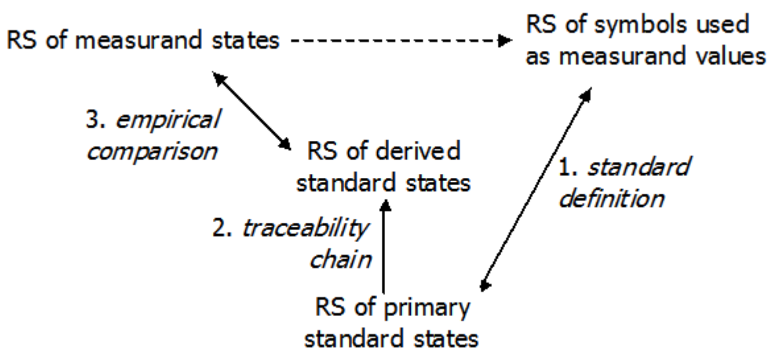


Figure 3 – Measurement as a scale-preserving evaluation obtained by the comparison to a standard derived by a primary standard.

Operations 1 and 2 are usually carried out *before* measurement: nevertheless measurement cannot be completed without them and therefore such operations play an essential role for the definition itself of measurement. As a consequence, measurement results must state a measurand value in reference to the adopted standard, usually expressed in the form of a measurement unit.

## 1.2. The output/input black box model

It is a well-known fact that different *methods of measurement* exist, each of them corresponding to a specific technique to perform the comparison between the measurand and the standard. While some methods require the synchronous presence of the measurand and the standard (e.g., following the paradigm of the two arm balance provided with a set of standard weights: a direct comparison) many others are based on the usage of devices acting as serializers of the comparison, so that a measurement involves (at least) two interactions: standard-instrument and measurand-instrument.

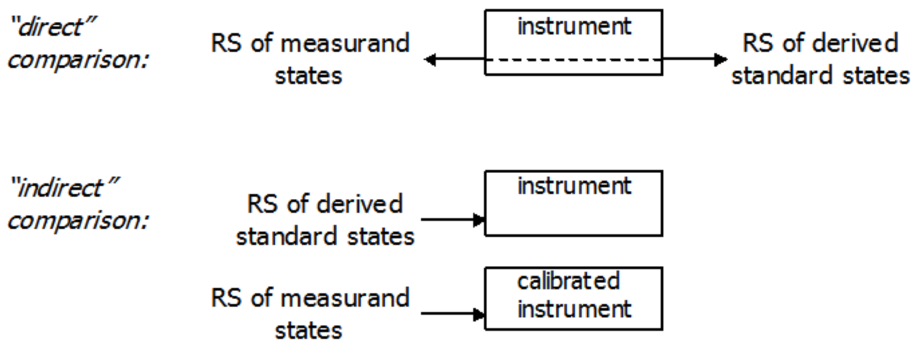


Figure 4 – The different usages of the measuring systems as comparators.

In its interaction with the measurand the instrument generates an output; a general problem of measurement can be then stated as follows: *from the output of the measuring instrument ("the reading") its input (the state of the system under measurement and its environment) must be reconstructed, and from this state a measurand value must be inferred*.

To cope with this input-from-output inference problem two basic strategies can be in principle followed:

- the analytical model of the measuring system behavior is identified and the obtained characteristic function is inverted, so that from the output readings the input signals are computed. Because of its complexity this approach is seldom adopted;
- the system is regarded as a black box and only its input-output behavior is taken into account: the instrument is put in interaction with a set of (known) standard states and the corresponding output readings are recorded; by a suitable interpolation this collection of couples becomes the so-called *calibration curve*, that can be thought of as a mapping from measurand values to instrument readings; this function is then inverted, so that each instrument reading can be associated with a measurand value.

The interactions standard-instrument and measurand-instrument have therefore a complementary function: while the former is aimed at creating a calibration diagram:

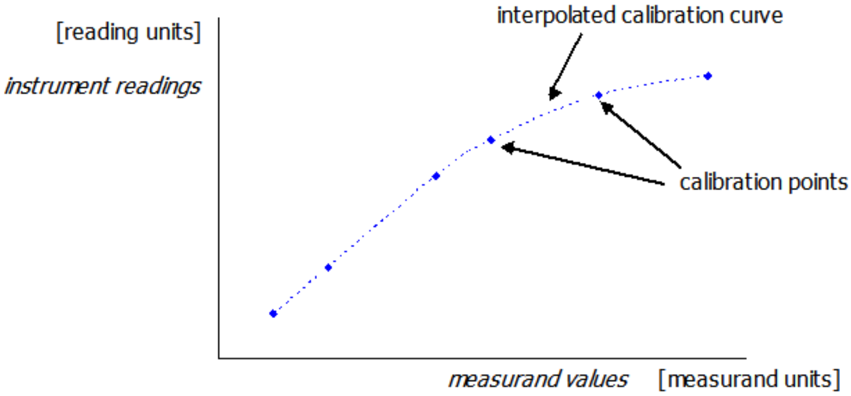


Figure 5 – A diagram with the example of a curve generated by calibration.

the latter uses the inverted diagram to find the measurand value that corresponds to the obtained reading:

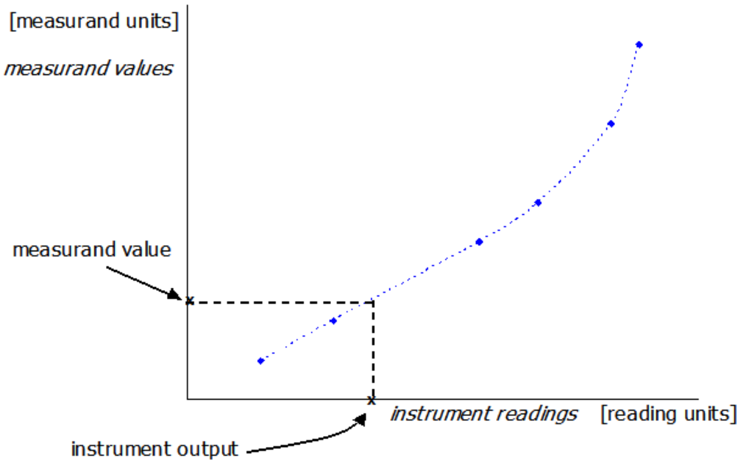


Figure 6 – The example of an inverted calibration diagram, for usage in measurement.

To enhance the user-friendliness of the measuring systems it is customary to set up their presentation component so that the data they display are expressed directly in measurand units, i.e., the calibration diagram is embedded into the systems. While always measurement requires calibration information, in these cases one can properly speak of *calibrated instruments*.

## 1.3. Set-theoretical model

The sensor behavior, therefore critical for both calibration and measurement, is usually expressed as a *characteristic function* formalizing the input-output conversion performed by the sensor itself. The sensor input, a couple $(x, \overline{w})$ where $x = x(t) \in X$ is the measurand and $\overline{w} = \langle w_1, \ldots, w_n \rangle = \langle w_1(t), \ldots, w_n(t) \rangle \in \overline{W}$ is a collection of further quantities influencing the sensor behavior, is transformed to its output $y \in Y$. Therefore the sensor characteristic function:

$$f : X \times \overline{W} \times T \to Y \quad (1)$$

takes the measurand $x(t)$, the influence quantities $\overline{w}(t)$ and the current time $t$, included to take into account possible time-dependent effects, and associate them with the output signal $y(t) = f(x(t), \overline{w}(t), t)$ to which both the measurand ("the signal") and the influence quantities ("the noise") contribute.

This simple formalization allows us to introduce some basic parameters describing the static behavior of a sensor:

- *sensitivity*: ideally $x_1 \neq x_2$ implies $f(x_1, \overline{w}, t) \neq f(x_2, \overline{w}, t)$, i.e., distinct measurand values always produce distinct outputs; the ratio $\Delta y / \Delta x$ expresses the aptitude of the sensor to reproduce measurand variations to output values;
- *selectivity*: ideally $f(x, \overline{w}_1, t) = f(x, \overline{w}_2, t)$ even if $\overline{w}_1 \neq \overline{w}_2$, i.e., the sensor output is not affected by the variations of influence quantities; the less is the variability of $y$ due to $\overline{w}$ the better is the sensor (therefore selectivity corresponds to non-sensitivity to influence quantities: the relative contribution of the measurand to the output can be formalized as a signal-to-noise ratio);
- *repeatability* and *stability*: ideally $f(x, \overline{w}, t_1) = f(x, \overline{w}, t_2)$ even if $t_1 \neq t_2$, i.e., the sensor output is not affected by short-term (fluctuations) and long-term (aging) time effects; the less is the variability of $y$ due to $t$ the better is the sensor (a stable sensor does not require frequent re-calibrations);
- *linearity*: ideally $y = ax + b$ (where $a$ and $b$ are given coefficients, possibly with $b = 0$), i.e., $f$ is a straight line, the better the actual sensor behavior is approximated by this equation the better is usually considered the sensor (a linear, zero-crossing sensor is calibrated in a single operation, aimed at determining the slope $a$).

In addition to these static parameters, the dynamic behavior of the sensor is synthesized by parameters such as its frequency response.

The technical specifications for sensors usually include some quantitative evaluation for these parameters in the nominal conditions of usage, expressed by the allowed ranges of measurand and influence quantities.

### 1.4. Generalized model

The inference process that leads to the evaluation and the expression of a measurand value is always only plausible in its results, and in general nothing can be inferred with certainty about the measurand value. The causes of this lack of certainty are various, and in particular:

- the model of the measurement system has not identified all the relevant influence quantities and one of them has a significant variability, such that the environmental conditions (including human operators) change after the calibration;

- the measuring system is less stable than expected when the calibration procedure was defined, i.e., the instrument would require a re-calibration before its usage;
- the interpolation shape of the calibration curve does not adequately map the actual instrument behavior (e.g., it is significantly non-linear where a piecewise linear interpolation was chosen), so that for some instrument reading subsets the instrument is wrongly calibrated.

All these cases can be formally characterized by recognizing that the certainty implied in the choice of a single-valued association between instrument readings and measurand values is not adequate: in the interaction with the measuring system during calibration, each measurand value generates an instrument reading that should be considered a sample drawn from a whole set of possible readings. Such variability can be formalized according to a set-theoretical model, so that the information obtained in the calibration is expressed by a *calibration strip*, in which an interval of possible readings, whose centre and width can be considered as the nominal reading and an uncertainty interval respectively, is associated with each measurand value:

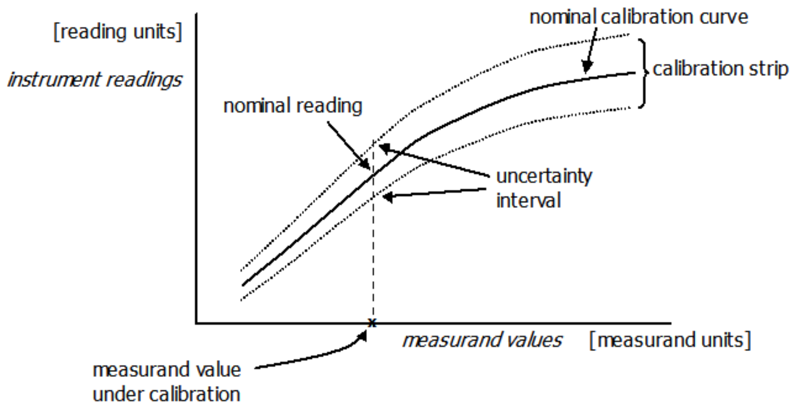


Figure 7 – A diagram representing a calibration in which uncertainty has been taken into account.

(note the changes of the calibration strip width along the measurand axis, taking into account non-uniformities in the uncertainty evaluation).

As in the previous (certain, and therefore ideal) case, this diagram is used in its inverted form during measurement: for any given instrument reading an uncertainty interval of possible measurand values is obtained together with a nominal value.

An even more general approach could be adopted by expressing the uncertainty estimation as a standard deviation, and therefore in a probabilistic framework, as recommended by the ISO *Guide to the Expression of Uncertainty in Measurement* (1993) (GUM). The *Guide*, based on a recommendation by the International Committee for Weights and Measures (CIPM) (1981), states that measurement uncertainty can be estimated on the basis of both statistical and non-statistical methods, and specifies a procedure to combine such components into a *combined standard uncertainty*}. The set-theoretical formalization can be then regarded as a specialization of this framework: if the combined standard uncertainty is multiplied by a coverage factor then an *expanded uncertainty* is obtained, thought of as the half-width of an uncertainty interval.

The inherent presence of uncertainty justifies the fundamental assumption that the result of a measurement must state not only a (nominal) measurand value but also its uncertainty estimation.

# 2. Characteristics and theory of knowledge

**Abstract -** *In the course of history the research about the nature of human knowledge, its characteristics and limitations has produced a huge amount of theories and philosophical systems. General topics such as the reality of the objects of knowledge, the relation between the objects of knowledge and their models, the possibility of a definitive foundation for knowledge have been differently interpreted by different philosophers and scientists, and they are schematically reviewed here.*

## 2.1. The problem of knowledge

Human beings know but do not definitely know what knowledge is: traditions, prejudices, expectations, and projections are more or less always part of knowledge and make it a combination of objectivity and subjectivity. Rationality allows some critical control on knowledge, but rational is the recognition of the limitations to which human knowledge is subject.

The interest in theorizing about knowledge arises from the observation that different persons have different beliefs, and ultimately that beliefs and facts are distinct: «theory of knowledge is a product of doubt», as Bertrand Russell wrote. In the history of both western and eastern culture such a doubt has stimulated an impressive amount of research, ideas, and philosophical systems, and nevertheless very different positions have been maintained and still remain on the nature of knowledge and its object (it is reasonable to hypothesize that, more than from the plethora of such positions, the complexity of the topic derives from its inherent reflexivity, due to the fact that the object of the analysis coincides with the tool by means of which the analysis is performed: to know knowledge only knowledge can be employed). A basic dichotomy can be identified, whose elements play the role of competing attractors for an ideal continuum of positions: *objectivism* assumes that a world external to the subject exists independently of him and has predefined properties, existing as such before they are acquired by the perceptive-cognitive system of the subject, whose aim is to reconstruct them; on the other hand, *solipsism* asserts that the cognitive system of the subject projects his own world out of him, and the reality of such a world is just an image of the laws internal to the system.

The position currently supported by the majority of scientists and engineers can be plausibly characterized as a kind of “pragmatic realism”, close to but not coincident with objectivism, according to which the conjoint efforts of science and technology are aimed at reaching, and actually guarantee, better and better, i.e., more and more objective, knowledge of the world whose properties are therefore progressively discovered.

Measurement plays a crucial role in supporting this realism.

## 2.2. The status of realism

In acquiring and processing information from the world human beings constantly *produce models* (and sometimes theories: we will not emphasize here the distinction between models and theories, grounded on formal logic) of the world they observe, thus generating knowledge on it.

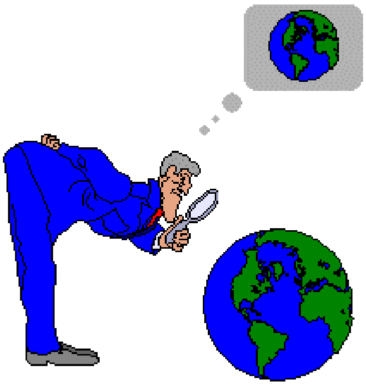

Figure 1 – Human beings produce modes of the world they observe

Such knowledge results from the relations among the three interacting entities, the subject, the world, and the model, so that the relation between the world and its models is not direct, but always mediated by the subject who produced the models themselves.

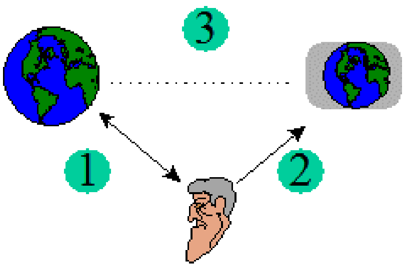


Figure 2 – The relations among subjects, world, and models

Whenever it remains individual, knowledge is just tacit and usually implicit and as such it reduces to personal experience that can be communicated only by person-to-person means, as imitation.

Critical is therefore the *objectivity*, i.e., the independence from the subject, of the relation between the world and its models. Realism assumes two operative reasons for justifying the possibility of some objective knowledge:

- *intersubjectivity*: were knowledge only subjective, mutual understanding would be an exception more than a rule;

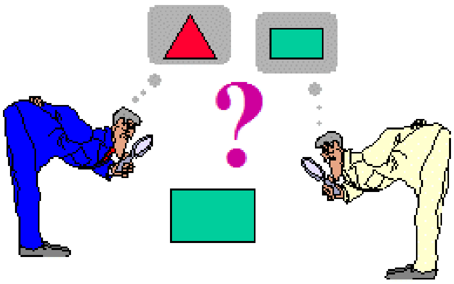

Figure 3 – A justification for realism: mutual understanding

- *pragmatics*: were knowledge only subjective, our ability to effectively operate on the world would be an exception more than a rule.

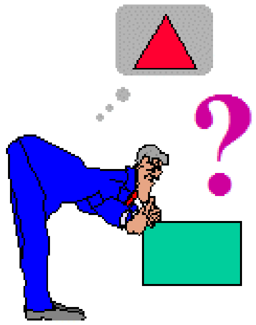

Figure 4 – A justification for realism: effectiveness

Realism can be then interpreted as a weak form of objectivism: world (exists independently of us and both intersubjective and pragmatic experiences lead us to assume that it) cannot be too different from our models of it.

On the other hand, to generate knowledge that can be shared subjective models must be expressed in some socially understandable and usable form, such as statements in a natural language or mathematical laws. This points out a further, basic, issue on knowledge: «how can it be that mathematics, a product of human thought independent of experience, is so admirably adapted to the objects of reality?», in the words of Albert Einstein.

Philosophers and scientists have formulated different opinions at this regard, more or less explicitly in reference to a basic dichotomy: either “scientific laws faithfully describe how the world is” or “scientific laws are just synthetic means to express information about events in an aggregate way”. The former position implies a *metaphysical* hypothesis on the nature of the world, classically stated as «numbers are in the world» (Kepler) or by assuming that «the great book of nature» cannot be understood «but by learning its language and knowing the characters in which it is written: it is written in mathematical terms» (Galileo); in contrast, the latter position suggests the *economic* nature of science: since «in Nature the law of refraction does not exist at all, but only different cases of refraction», by means of such a law «we do not have to keep in mind the countless phenomena of refraction in the various compositions of matter and under the various incidence angles, but only the rule that we call “law of refraction”, a much easier thing» (Mach).

Measurement has been often adopted to justify the former position.

## 2.3. Semiotics of knowledge

Knowledge can be *about* physical world but it is not *part of* it. Given the realistic assumption of the independence of the physical world from the subject, both subjective and objective knowledge can be interpreted in an evolutionary context as the results of mankind to adapt to his (firstly only physical and then also social) environment. At this regard Karl Popper has suggestively proposed to identify «some stages of the cosmic evolution» as organized in three “worlds”, as follows:

World 1

0. Hydrogen and helium

1. Heavier elements; liquids and crystals
2. Living organisms

World 2

3. Sensitivity (animal conscience)
4. Conscience of self and death

World 3

5. Human language; theories of self and death
6. Products of art, technology, and science

In this framework knowledge (whose object can belong to either Worlds, and finally could even become knowledge itself…) is a rather advanced entity, appearing initially within World 2, in the form of subjective experiences, and then fully evolving in the context of World 3. The transition from World 2 to World 3 corresponds to the social ability to communicate, and therefore to share, experience: that is why the availability of a (textual or non-textual) language is considered the first step within World 3. Furthermore, the usage of a language gives knowledge a syntax and make it a semiotic entity.

Given the complexity of the concept of knowledge and its fuzzy characterization, rather than trying a definition of it we suggest that the (possible) presence and the relative importance of the semiotic components, syntax, semantics, and pragmatics, can be adopted as a criterion to distinguish among the different entities that are commonly deemed to be (related to) knowledge. In particular:

- the exclusive availability of *pragmatic* information ("to know how to do"), such as the competence shown by many craftsmen, appears to be a limited kind of knowledge, if knowledge at all;
- the exclusive availability of *syntactical* information and the ability of purely symbolic (i.e., only syntactical) processing, as performed by most automatic devices, appears to be a limited kind of knowledge, if knowledge at all.

The designation of "knowledge-based" for the systems operating on the basis of an explicit semantics is a further argument to support the hypothesis that meanings are critical for the emergence of "proper" knowledge, and therefore that socially communicable knowledge ("World 3 knowledge") is an entity spanning all the semiotic components.

## 2.4. Pragmatic classification of models

If the pragmatic component is taken into account, different purposes for knowledge can be recognized: models can be adopted for description, explanation, prevision, prescription.

It is usual that the first stages of the development of a new field of knowledge are devoted to the production of models aimed at the *description* of the system under analysis. Typical outcomes of this work are the identification of properties relevant to describe the system and their evaluation to classify the system itself into more or less rough categories.

To overcome the conventionality of taxonomies and whenever the available knowledge allows it, some relations among properties are identified, so that each property is embedded in a network of dependencies. In such cases the relational information that is (explicitly or implicitly) conveyed by properties can be referred to in order to obtain an *explanation* of the system state / behavior: the value of the property $x_1$ is $v_1$ because $x_1$ is connected to the properties $x_2,...,x_n$ by the relation $R$, and the properties $x_2,...,x_n$ have values $v_2,...,v_n$ respectively, and $R(x_1,...,x_n)$.

Sometimes models can be further enhanced to include relations connecting properties with an explicit functional time dependence, $\forall i=1,...n, x_i = x_i(t)$, for example in the form (known as *canonic representation*, or *local state transition* in System Theory):

$$\frac{dx_i(t)}{dt} = f_i(x_1(t),...,x_n(t)) \tag{1}$$

for time-continuous models, and:

$$x_i(t+\Delta t) = x_i(t) + f_i(x_1(t),...,x_n(t))\Delta t \tag{2}$$

for time-discrete models. Models can be then used also for *prevision*, in particular if the integral / time-global versions of the canonic representations is taken into account:

$$x_i(t) = x_i(t_0) + \int_{t_0}^{t} f_i(x_1(\tau),...,x_n(\tau))d\tau \tag{3}$$

$$x_i(t_n) = x_i(t_0) + \sum_{j=0}^{n-1} f_i(x_1(t_j),...,x_n(t_j))\Delta t \tag{4}$$

allowing to compute the system state $< x_1(t),...,x_n(t) >$ at a generic (future or past) time $t$ from a reference, initial state $< x_1(t_0),...,x_n(t_0) >$ and by means of the state transitions $< f_1,..., f_n >$.

Finally, if an external intervention is possible on the system, its spontaneous dynamics can be controlled to let the system evolve toward a required target. In this case, models are then aimed at *prescription*: given a generalized version of the local state transition function including in its domain both the current state and the user input, models specify how to provide such an input, and therefore become decision-making tools.

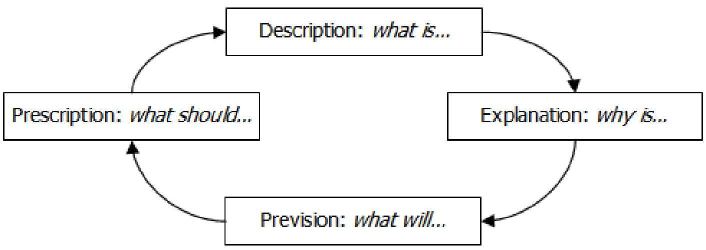


Figure 5 – The knowledge loop among the four kinds of models

According to the traditional paradigm of science and its relations with technology, by repeatedly following this *knowledge loop* the quality of knowledge itself and the effectiveness of system control can be always enhanced.

On the other hand, in many situations prescriptions are required even when predictive, explanatory, and sometimes even socially agreed descriptive models lack (let us quote the crucial examples of medicine and business administration). In these cases experiences and expectations (i.e., World 2 knowledge) still play a critical role.

## 2.5. The evaluation of quality of knowledge

Given the combination of subjectivity and objectivity so usually present in knowledge, it is not amazing that the evaluation of the quality of knowledge represents a basic issue in the process of acquisition of candidate items for their integration in an existing body of knowledge.

The quality of a model can only be evaluated in reference to the goals for which the model itself has been produced: the general criterion for this quality evaluation is therefore the *adequacy to goals*. Truth, traditionally thought of as "correspondence to facts", is regarded as a specific case of adequacy, applicable whenever correspondence to facts is indeed considered an important issue (note how this position radically differs from the pragmatist definition of truth, according to which «a sentence may be taken as a law of behavior in any environment containing certain characteristics; it will be "true" if the behavior leads to results satisfactory to the person concerned, and otherwise it will be "false"» (Russell)).

While adequacy is hardly object of a general treatment, the possibility of evaluating the truth of a model has been widely debated and is surely one of the most critical topics of Philosophy of Science. Following Karl Popper, it can be suggested that the controversy is specifically related to two basic Problems:

1. how to compare (the statements of) competing models?

2. how to evaluate (the truth of) a model?

in reference to which three standpoints can be identified:

- *verificationism*, typical of classical science and brought to its extreme consequences by the Neo-Positivistic school: the Problem 2 admits a solution (and therefore the truth of a model can be determined), from which a solution to the Problem 1 is derived: the reference to truth is the foundation allowing the advancement of science;
- *falsificationism*, also called "critical rationalism", as advocated by Popper himself: the Problem 1 admits a solution (in presence of competing models the one is chosen that is not falsified and has the greater empirical content), but a solution to the Problem 2 cannot be derived from it: by means of conjectures and confutations truth is approximated; the preference of a model over a competing one can be rationally motivated, but a model cannot be justified in itself;
- *epistemic relativism*, also called "irrationalism", supported by philosophers such as Thomas Kuhn and, in its extreme consequences, Paul Feyerabend: the Problem 2 does not admit a solution («the only principle that does not inhibit progress is: anything goes. For example, we may use hypotheses that contradict well-confirmed theories and/or well-established experimental results. We may advance science by proceeding counter-inductively» (Feyerabend)), and therefore also the Problem 1 cannot be solved: no criterion / method that is absolutely valid holds in scientific research.

## 2.6. Data and inference in knowledge

We get an insight into knowledge by considering its operational side of being a *faculty to solve problems*, and in particular to modify the state of systems according to given goals. As human beings we constitutively have the ability to operate state transitions on the systems with which we interact by means of a "World 2 strategy": we acquire data on the current state through our sensorial apparatus; by means of brain we perform inference on such data, and finally we use the data resulting from this process to drive our motor apparatus whose activity actually carries out the required state transition (this three steps correspond to the tripartite structure of the neural system: *sensorium*, brain, and *motorium*; note that more than 99% of the about $10^{10}$ neurons of human beings are part of the brain). In many cases this strategy is manifestly both more efficient and more effective than a blind "try-and-error" approach, although far more complex than it.

The same conditions, the availability of data and the ability to deal with them by means of inference, are also characteristic of the "World 3 strategy" to problem solving:

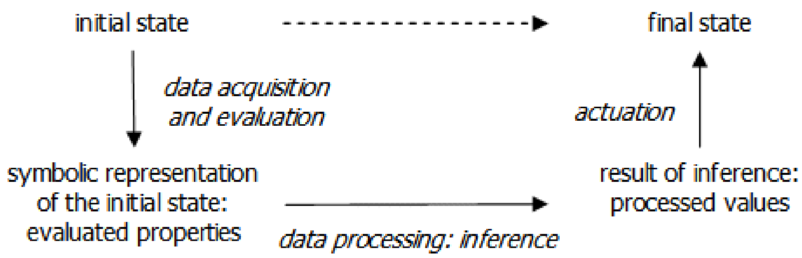


Figure 6 – The "World 3 strategy" to problem solving

This scheme highlight the complementary role of data and inference in knowledge.

*Data*, i.e., evaluated properties, are aimed at being *faithful representative* of the observed state, as obtained by either subjective or inter-subjective and objective procedures, and can be expressed by means of either an informal or a formal language. The fundamental operation to empirically get formal data by means of an inter-subjective and objective procedure is measurement: according to the representational point of view to measurement theory, such a faithfulness is formalized by requiring that the mapping from empirical states to symbols be a homomorphism for the scale type in which states are measured; the existence of monomorphisms for the measurement scale type (i.e., admissible scale transformations) manifests the residual presence of conventionality in the selection of symbols. It is at this regard that one could wonder about *the truth* of symbols and the related sentences.

*Inference* is an operation aimed at obtaining new data ("conclusions") from the processing of the given inputs ("premises"). To understand the structure of inferential processes the fundamental distinction between *singular* and *universal assertions* (sometimes called *facts* and *laws* respectively) must kept into account. In set-theoretical terms, $a \in P$ (the element $a$ belongs to the set $P$; the property $P$ holds the element $a$) is singular, whereas $P \subset Q$ ($P$ is a subset of $Q$; for all elements $x$, if the property $P$ holds for $x$ then also the property $Q$ holds for it) is universal (it should be clear therefore that data obtained by means of measurement are singular).

Two kinds of inference are then traditionally considered, that in their simplest forms are as follows:

- from the singular $a \in P$ and the universal $P \subset Q$ by *deduction* the singular $a \in Q$ is obtained; deduction is a truth-preserving inference that, strictly speaking, *does not lead to new knowledge*;
- from a collection of singular $a \in P$ and $a \in Q$ by *induction* the universal $P \subset Q$ can be obtained; induction is a hypothetical inference that *does not lead to conclusive knowledge*.

The problem of foundation of empirical knowledge is traditionally ascribed to this circularity: deduction leads to true conclusions, but only if the truth of its (both singular and universal) premises can be assumed; induction is the only means to obtain new universal knowledge, but the truth of such a knowledge cannot be definitely assumed.

## 2.7. Non-exactness of knowledge and measurement

We have already noted the relevance of language for World 3 knowledge: truth is a property of sentences (actually: of declarative ones), and «science, though it seeks traits of reality independent of language, can neither get on without language nor aspire to linguistic neutrality. To some degree, nevertheless, the scientist can enhance objectivity and diminish the interference of language, by the very choice of language» (Quine). That is why formalization (i.e., the expression of knowledge in a form such that inferential processes can be entirely performed on the basis of the syntactical component of data) is often regarded as a critical requirement for scientific knowledge. On the other hand, formalized languages can be (and in many cases actually are) far too precise for expressing empirical knowledge: «there are certain human activities which apparently have perfect sharpness. The realm of mathematics and of logic is such a realm, par excellence. Here we have yes-no sharpness. But (...) this yes-no sharpness is found only in the realm of things *we say*, as distinguished from the realm of things *we do*. (…) Nothing that happens in the laboratory corresponds to the statement that a given point is either on a given line or it is not» (Bridgman).

Hence the same empirical knowledge can be expressed in sentences by balancing two basic components: *certainty* (a term for some aspects more general than truth) and *precision* (also called specificity or, at the opposite, vagueness). Therefore «all knowledge is more or less uncertain and more or less vague. These are, in a sense, opposing characters: vague knowledge has more likelihood of truth than precise knowledge, but is less useful. One of the aims of science is to increase precision without diminishing certainty» (Russell).

The fact that the length of the diagonal of a physical 1 m side square *cannot* be $\sqrt{2}$ m is an important consequence of metrological thinking: the information conveyed by real numbers (and the related concepts of continuity / differentiability) is too specific to be applicable, as is, to physical systems. By progressively enhancing the resolution of the measuring systems, and therefore by increasing the specificity of the measurement results, their uncertainty consequently grows, until the object of measurement itself becomes uncertain (in the previous example, at the atomic scale the concept of “physical square” is meaningless), and an “intrinsic uncertainty” (also called “model uncertainty”) is reached.

This reflects a basic feature of the relation that by means of knowledge it is established between World 1 (to which the object of knowledge belongs) and World 3 (to which the sentence that expresses knowledge belongs): if symbols are not generally so specific to univocally denote (properties of) things (2+2=4 holds for both apples and aircraft carriers), at the same time things are too complex to be fully described by means of symbols.

## 2.8. (Non-)foundations of knowledge

Philosophy of Knowledge (and Philosophy of Science in particular) has always quested for a *foundation* of knowledge, i.e., the elements on which the "building of knowledge" can be firmly erected. In the course of history such a foundation has been found in natural elements (for example Thales of Miletus affirmed that the principle that causes all the things is water, while Heraclitus of Ephesus found it in fire), in physical or conceptual structures (atoms according to Democritus of Abdera, numbers in the conception of Pythagoric school), in metaphysical principles (such as the hypothesis that Nature is simple), in methodological assumptions (in particular the postulation that any empirical knowledge cannot derive but from sense data). The usage of the metaphor of foundations is not conceptually neutral: the architectural image of "foundations" reveals the hypothesis that scientific research can make knowledge incrementally grow from its bases, where measurement has been traditionally recognized as the operation able to produce the objective data playing the role of such bases.

In the last decades this confidence on the progressive development of scientific knowledge has been questioned by concentric objections, all emphasizing that definitive foundations are beyond the reach of the means human beings adopt to know. Complementary to the above mentioned philosophical positions of epistemic relativism (according to which raw sense data do not exist because data are always theory-laden), an important area of scientific research is currently devoted to the systems that exhibit relevant *structural complexity*, a characteristic that makes such systems irreducible to the classical paradigm of reduction to simplicity through the hypotheses of linearity, principle of superposition of effects, …

Knowledge is recognized to be an always evolving process, where «there is never an absolute beginning. We can never get back to the point where we can say, "Here is the very beginning of logical structures."» (Piaget). More than the actual availability of data, knowledge is recognized to be a potentiality (what is "stored" in our brain is how to compute multiplications, not the results of operations such as $1234 \times 5678$), information always under reconfiguration.

The role assigned to measurement is paradigmatic of the shift towards what could be called *reticular* (and therefore without foundations) knowledge. Indeed, according to the current standpoints of philosophy of measurement:

- since measurement results depend on standards through a traceability chain, standards themselves could be thought of as "realizations of true values", then playing the role of actual foundations for measurement; on the other hand, standards must be indeed "realized" by primary laboratories, who maintain their quality by means inter-laboratory comparisons: therefore this claimed "path towards foundations" cannot but include a component of conventionality;
- any measurement result depends for its evaluation on the previous measurement of a set of influence quantities, in their turn being new measurands so that in principle such a dependence should be recursively applied, with the consequence that a "well founded measurement" would be impossible to be completed; the usual operative choice to assume that the quantities influencing the initial measurand are not influenced by other quantities, and therefore that they can be "directly measured", highlights the conceptual approximation inherent to any measurement;

- while the adequacy of empirical models is controlled by means of measurement, the quality of measurement results depends on the quality of mathematical models used for designing measuring systems.

This complexity makes knowledge the most versatile tool available to human beings and a fascinating object for knowledge itself.

# 3. Principles of semiotics as related to measurement

**Abstract** - *Semiotics investigates the symbolization, as related to the coding and the decoding of information in a system of signs. The relation of "standing for" is introduced and analyzed here in its elements of conventionality, in particular in reference to the opposition between analogue and digital coding, and its systemic nature is discussed in terms of the classical distinction of syntax, semantics, and pragmatics. Finally, measurement is presented as a peculiar semiotic operation.*

## 3.1. Signs as entities that "stand for" something

Semiotics is commonly defined as the *doctrine of signs*, a sign being «something which stands to somebody for something in some respect or capacity» according to Charles S. Peirce, one of the seminal thinkers about semiotics itself. The emphasis is here on the relation of "standing for", which in its simplest form can be modeled as follows.

When a purpose is assigned to, or recognized proper of, things, they can be evaluated in their ability to satisfy it. Any given purpose induces a relation of *functional substitutability S* on the set of considered things *T* such that $\forall x, y \in T,\ S(x, y)$ if and only if *x* is a substitute of *y* with respect to the purpose, i.e., *x* is as able as *y* to satisfy the purpose itself.

Instead of investigating here the general properties of the relation *S* (but at least the observation should be made that in many cases functional substitutability is not a matter of a yes-no alternative, and therefore that *S* could be usually formalized as a fuzzy relation), let us devote our attention to the specific relation of *identification*. The functional substitution implied in the identification is such that *x* identifies a given *y* if and only if *x* operates as the selector of *y* in a set of candidate things $y_1$, $y_2$, … the exhibition of *x* being considered functionally equivalent to the selection of *y* and the non-selection of any other $y_i \neq y$ in the candidate set. In such a relation $S_{id}(x,y)$ let us call *x* and *y* the *identifier* and the *identified entity* respectively: *x stands for y*. For example, the sound c-h-a-i-r (a physical thing as a space-time event produced by the utterance of a speaker) could be the identifier chosen to select a chair instead of any other non-chair object (note that no restrictions have been imposed on the set of the entities which are object of identification: *y* can be a physical object but also an informational entity).

It is a common observation that different $x_1$, $x_2$, … can be adopted as identifiers for the same entity *y*, $S_{id}(x_1,y)$, $S_{id}(x_2,y)$, … (a chair can be identified by different sounds, possibly pronounced by different persons in different languages in different times, but also by writings, drawings, gestures, …). In this case the $x_1$, $x_2$, … are functionally substitutable with each other in their role of identifiers for *y*, and therefore a derived relation $S_{id\text{-}y}$ holds among them. Formally (we will continue to forget the fact that also $S_{id\text{-}y}$ could be fuzzy) $S_{id\text{-}y}(x_1,x_2)$ if and only if $S_{id}(x_1, y) \wedge S_{id}(x_2, y)$.

A fundamental step is taken when the *class* $\hat{x}$ of all the identifiers $x$ for which the relation $S_{id\text{-}y}$ holds is abstractly considered as the identifying entity for $y$, thus recognizing that signs, although instantiated in physical things, are *information entities*.

## 3.2. Coding and decoding

Signs generally result from the relation between two elements: identifiers and identified entities. Such a relation is operatively realized and performed in two phases:

- for a given entity $y$ to be identified, an identifier is obtained by means of an operation of *coding*: an information entity $\hat{x}$ is at first associated with $y$, and then an identifier $x$ is selected such that $x \in \hat{x}$; for example:

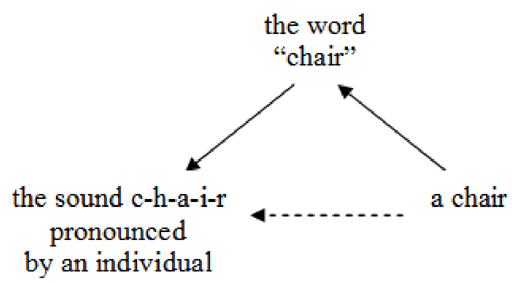


Figure 1 – An exemplification of the structure of a coding operation

- for a given identifier $x$, the identified entity is obtained by means of an operation of *decoding*: an information entity $\hat{x}$ is at first identified, by means of a pattern recognition, as the class to which $x$ belongs, and then an entity $y$ is selected as associated with $\hat{x}$; for example:

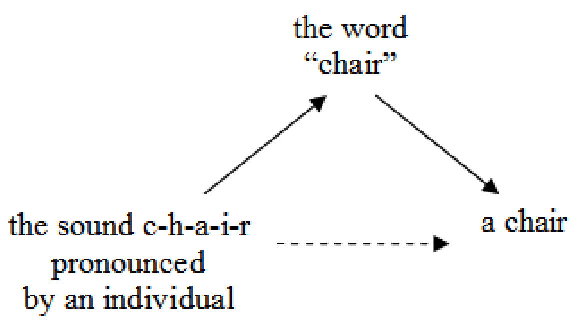


Figure 2 – An exemplification of the structure of a decoding operation

The previous two diagrams are instances of a more general "semiotic triangle", in which the "standing for" relation is represented as follows:

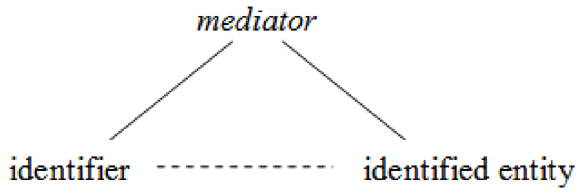


Figure 3 – The “semiotic triangle”

where the dotted line expresses that the relation is indirect and a “mediator” is usually required to connect identifiers and identified entities.

In the history of Semiotics diagrams of this kind have been widely adopted to present and generalize the relation that we have introduced as between identifiers and identified entities. For example, F. de Saussure defined it in terms of “signifiers” and “signifieds” (and called “signification” the relation itself), while L. Hjelmslev used the terms “expression” and “content” respectively.

These diversities witness the different interpretations and emphasis put on the elements of the relation. For example, the mediator has been thought of as either the *sense* of the identifier (chairs are identified by means of the term “chair” because of the meaning associated with such a term, i.e., the set of features which are shared by everything to which the term applies, the so-called *intension* of the term), or the *set of the entities* the entities to what the identifier stands for (the so-called *extension* of the term), or the *subject* with the competence to maintain the relation, or the *social context* of the individuals who agreed to associate the identifier with the identified entity.

In the case the entity to be identified belongs to the physical world (let us mention again that it could be a purely informational entity, as in the case one is talking about words or numbers) a common, although surely not necessary, situation is such that the relation between the physical thing chosen as identifier and the identified entity is mediated by *two* informational entities, for example:

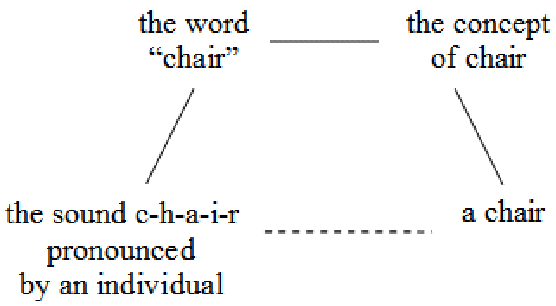


Figure 4 – The structure of the semiotic relations

so that each arrow in the diagram :

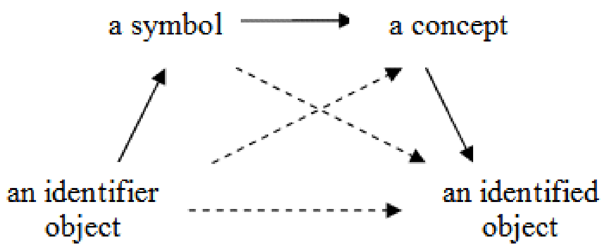


Figure 5 – The “standing for” relations

represents a partial realization of the “standing for” relation.

Semiotics has been particularly working on the *informational component* of the “standing for” relation, therefore often minimizing, or even neglecting, the analysis of the relation between the symbols and the physical things adopted as support for them. From now on we will accept this general standpoint, and follow the terminology proposed by C. Ogden and I. Richards who describe the “standing for” relation in terms of *symbols* that stand for *referents*.

## 3.3. Conventionality of signs

The “standing for” relation is a complex one: the same symbol could stand for different referents, and different symbols could stand for the same referent (e.g., in the case of the linguistic phenomena of *polysemy* and *synonymy* respectively). This suggests that such a relation is not inherent to the entities involved in it: an entity becomes a symbol only as the result of a choice.

The issue of the (degree of) arbitrariness of signs has been thoroughly inquired by many philosophers, who noticed its fundamental implications in terms of autonomy of symbols in relation to referents and therefore, generally speaking, of language in relation to reality. For example, in Plato’s Cratylus the problem of “right names” for things is discussed, and it is concluded that «whatever name you give to a thing is its right name; and if you give up that name and change it for another, the later name is no less correct than the earlier, just as we change the name of our servants; for I think no name belongs to a particular thing by nature».

Following Peirce, signs are usually distinguished in three broad categories, characterized by their increasing degree in conventionality:

- *indexes*, such as “natural signs” (smoke standing for a yet unseen fire) and “signals” (a phone ringing standing for a waiting call), for which the symbol is causally connected to the referent, so that every subject informed on the connection is able to infer the existence of the relation;
- *icons*, such as images or onomatopoeic words, for which the relation symbol-referent is based on some mutual resemblance or imitation of the related entities; in this case the relation can be intensively learned (e.g., the higher the sound the angrier the speaker) and is easily, while often implicitly, shared among cultural communities;
- *symbols* (in specific sense), such as those constituting many elements of textual languages, for which the relation symbol-referent is purely conventional (obtained sometimes by an

explicit ruling convention, sometimes by usage), so that it must be learned according to an extensive strategy, i.e., by explicitly listing the symbol-referent pairs.

## 3.4. The opposition analogue / digital in a semiotic perspective

The latter two categories, icons and symbols, can be meaningfully expressed in terms of the *opposition between analogue and digital*, as traced back to the concept of *structure* modeled and formalized in Measurement Theory. The opposition A/D concerns the strategy adopted for *coding and decoding the meta-information* that complements the information that symbols convey on referents. Indeed, together with the information enabling the selection of referents, in many cases some structural information must be maintained in coding and recognized in decoding. For example, if the referent is a grade in {A,…,E} the observation of a physical support coding the symbol "B" should bring both the information "is B" (and "is not A, and not C, …") and the (ordinal) meta-information "is less than A but more than C, …". Therefore:

- *analogue* is the strategy by which the meta-information is coded in the support configuration, so that both coding and decoding correspond to the application of a homomorphic (i.e., structure preserving) mapping;
- *digital* is the strategy by which the meta-information is maintained in the coding rule, so that the physical support is only required to be able to assume at least two distinguishable configurations (those usually symbolized as "0" and "1"), as specified in Shannon's Theory of Information.

This characterization accounts for the nature of *opposition* of the strategies of analogue and digital coding:

- the definition of the code rule can be intensive in analogue cases, whereas must be extensive in digital cases: while analogue codes can be analytically defined, the lack of structure forces digital codes to be defined by explicitly and completely listing the occurrences symbol-referent;
- the set of information entities to code can be non completely pre-identified in analogue cases, whereas must be pre-identified in digital cases: for example, to increase the cardinality of the set of the possible symbols to code on a physical support an analogue code can be adopted as is, whereas a digital code must be redefined.

On the other hand;

- analogue coding can be adopted only if some meta-information is available, whereas digital coding is always available for finite sets of symbols;
- supports adopted in coding must be able to maintain the meta-information in physically distinguishable configurations in analogue cases, whereas can be very simple since only two distinguishable configurations are in principle required in digital cases.

It should be noted that mixed (partly analogue, partly digital) coding rule are common, as in the case of the usual numerical notation, in which the single digits are digitally coded while the positional rule is analogue.

### 3.5. The systemic nature of signs

The “standing for” relations are seldom defined as single and independent coding rules. Rather, their conventionality is considerably restricted by the effects derived by their *systematic definition* (as an example, consider the possibility of reconstructing the meaning of a linguistic term, i.e., “decoding” it, by means of its etymological analysis).

Natural languages, such as English or Italian, are far more complex than artificial languages, such as the formalism of mathematical logic or computer programming languages, also because they include a huge amount of exceptions, i.e., irregularities, in their coding and decoding rules. Nevertheless, the fact that some systematic effects progressively emerge from historical usage instead of explicit decision, as indeed in the case of natural languages, does not reduce their relevance but only the uniformity of the system of rules.

This systemic component was called *language* (*langue*, in French) by Saussure, who contrasted it with *speech* (*parole*, in French), regarded as the individual act of selection and actualization of symbols that stand for intended referents by means of some coding rules of the language. Any specific film would be therefore the “speech” of the “language” of cinema, an example highlighting that *langue* / *parole* is actually the dichotomy code / instance or schema / usage, as Hjelmslev termed it. «Each of these two terms achieves its full definition only in the dialectical process which unites one to the other: there is no language without speech, and no speech outside language: it is in this exchange that the real linguistic praxis is situated» (Barthes).

Dialectical is also the process by which the decoding of composite structures of symbols, e.g., sentences, is performed: paradigmatically, whenever coding rules are *context-sensitive* (a typical characteristic of natural languages), not only the meaning of a sentence is derived from the meaning of its constituting parts, but also the meaning of such parts could depend on their role in the sentence, so that it can be determined only after some hypothesis of the meaning of the whole sentence itself. This generally makes the recognition of the “standing for” relations a complex, recursive process, and again this explains why the constructs of artificial languages are defined as context-free whenever recognized as adequate (as an example of the role of context in rule evaluation, consider two possible definitions of the disjunction operator OR: if $v(x)$ is the truth value of the sentence $x$, in classical logic such an operator is context-free, since $v(x \vee y) = f(v(x), v(y)) = \max(v(x), v(y))$; on the other hand, in the case of probabilistic logic $v(x \vee y) = v(x) + v(y) - v(x \wedge y)$ and therefore the operator is context-sensitive, because $v(x \vee y) \neq f(v(x), v(y))$, with the term $v(x \wedge y)$ playing the role of context).

### 3.6. Syntax, semantics, and pragmatics

A fundamental classification to isolate the different contributions to the complexity of the “standing for” relation was proposed by Charles W. Morris, who suggested three basic sub-disciplines as the constituting components of Semiotics: syntactics (also, and more commonly, called syntax), semantics, and pragmatics. Despite their large reciprocal autonomy, such disciplines can be presented in terms of progressive extension of scope:

- *syntactical* is the information dealt with as *data*, taking into account the collection of available signs and its structure; a basic issue of syntax is *parsing*, i.e., the check that a sentence is well formed according to a given set of syntactical rules;
- *semantic* is the information dealt with as data provided *with meaning*, taking into account (also) the entities to what the signs stand for; a basic issue of semantics is *truth evaluation*, i.e., the check of the correspondence between the content of a sentence and the actual state of the reality (it should be noted that the evaluation of the truth of a sentence does not always require the interpretation, i.e., the "semantization", of the sentence itself; for example, tautologies in propositional logic (e.g., $A \vee \neg A$) are true for any interpretation of *A*; this establishes a distinction between *linguistic* and *empirical truth*);
- *pragmatic* is the information dealt with as data provided with meaning *and value*, taking into account (also) the relation of signs to interpreters and therefore issues related to behaviors, subjective interests, utilities, …; a basic issue of pragmatics is *relevance assessment*, i.e., the check that a sentence is actually useful for its deemed receivers.

The distinction among these disciplines and their goals can be exemplified by means of that particular system of signs that is mathematics:

- the formula "$2)\sqrt{+\leq x}$" is not well-formed, so that a fortiori neither its meaning and truth nor its utility for a given subject can be evaluated;
- the formula "2+3=4" is well-formed, its meaning can be evaluated and is actually false in the usual interpretation of its constituting signs;
- the formula "1=1" is well-formed and is true, but plausibly useless for most subjects.

Given the centrality of the "standing for" relation, semantics can be considered the core component of Semiotics. Nevertheless, the threshold between syntax and semantics is not always well defined, and often actually a matter of "point of view". For example, in the case of Morse coding the physical support is an electric current, whose patterns are interpreted as sequences of "dots" and "dashes", whose patterns are in their turn interpreted as sequences of alphanumeric characters; the sequence "dot-dash" is then a semantic entity with respect to an electric signal but it is a syntactical entity with respect to its deemed interpretation, i.e., the character "a".

### 3.7. Semiotics and communication

An important area of application of Semiotics is *communication*, i.e., the transfer of messages conveying some sort of information from a sender to a receiver through a channel. The standard model for a basic communication system has been defined by Shannon: the message generated by the sender is coded into a pattern of signs, here called a signal, that is transmitted by the channel and finally decoded again into a message for making it accessible to the receiver.

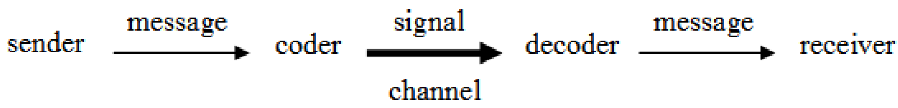


Figure 6 – The structure of a communication system

As formalized by Shannon, the communication problem – how to maximize the probability that the received message is the same as the one generated by the sender even in presence of a noisy channel – specifically relates to syntax. The general semiotic nature of the problem has been shown by Roman Jakobson, who suggested that each of the six components of a communication system:

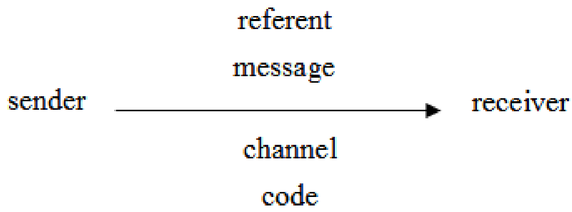


Figure 7 – The components of a communication system

is associated with a specific *function*, more or less present and emphasized in each communication act. Indeed, if the prominence is usually given to the referent (i.e., the "content" of the message, correspondingly to the so-called *referential*, or *cognitive*, *function*), in many messages the role of the other functions must be also taken into account to fully understand the communication act itself. According to Jakobson, a communication can differently stress:

- the position of the sender (*emotive function*) on the communication subject (e.g., in terms of rage or irony);
- the orientation towards the receiver (*conative function*), as typically in the case of imperative sentences, whose aim is indeed to convey commands to receivers;
- the role of the channel (*phatic function*), whenever a support to the management of the communication itself is required, for example to check whether the channel is still operative between the sender and the receiver (e.g., "are you still there?");
- the formal structure of the message itself (*poetic function*), for example when homophonies, rhymes, … are adopted;
- the requirements on the code (*metalinguistic function*), typically whenever the sender and the receiver want to check whether they are using the same coding rules (e.g., "what do you mean?").

The semantic component is critical in the communications with a prevailing referential function, and the problem of the truth evaluation of their contents can be generally posed. The other functions are instead oriented to the pragmatics of the communication: messages such as "fantastic!", or "excuse me", or "repeat please", or … are conveyed to obtain some communicational aim more than to state a meaning.

## 3.8. Applying semiotic principles to measurement systems

As an operation aimed at expressing in symbols the information empirically obtained on a system about a quantity, measurement can be meaningfully analyzed in its semiotic character. In comparison with other forms of judgment, there are two general grounds of peculiarity for measurement:

- the mediator between the referent (i.e., the measurand) and the symbol (i.e., the measurement result) is an empirical entity, external to both the measured thing and the measurer subject: the measurement system;
- the symbols adopted as identifiers for the measurands are chosen in a formal language, whose composition and structure are explicitly known.

While for a general semiotic system only the syntactical component can be usually formalized (the attempt was done by logics and philosophers of science such as Y. Bar-Hillel, R. Carnap, and J. Hintikka to quantify the semantic information conveyed by propositions as their "quantity of content": with such a broad connotation, the problem remained largely unsolved and was substantially left aside), these characteristics of measurement allow to consider it in some more specific terms:

- from the *syntactical* point of view: measurement can be thought of as an operation of selection of a symbol from a set, the actual granularity of such a set (as usually formalized in terms of either number of significant digits or expanded uncertainty) depending on the resolution of the sensing device; the usual Shannon's concept of quantity of information can be adopted in this case, such that the quantity of information conveyed by a measurement result increases as its uncertainty decreases;
- from the *semantic* point of view: measurands are always evaluated relatively to a reference, that is explicitly reported in measurement results in terms of a measurement scale (and specifically measurement unit whenever applicable) and that expresses the actual meaning for the (usually) numerical symbols by which the measurand is quantified; each measurement scale is characterized by a type, the most common scale types (nominal, ordinal, interval, ratio, absolute) being linearly ordered according to the algebraic structure they imply on the symbol set; the degree of semantic information conveyed by a measurement result depends thus on the degree of richness in algebraic structure of its scale type (formally, the semantic information increases as the class of admissible transformations for the scale type becomes more specific);
- from the *pragmatic* point of view: because of the existence of functional relations connecting them (the typical case of physical laws), measurands are embedded in a network of pragmatic information allowing to obtain new measurement results by computation, i.e., by derived measurement; while a completely disconnected measurand can be defined in a totally arbitrary way, and therefore its evaluation is pragmatically useless, the more the

measurand is connected (e.g., the greater is the number of functional relations in which it is present), the higher is the degree of pragmatic information conveyed by its values.

Even this summary presentation shows how the semiotic perspective can be useful to understand some fundamental characteristics of measurement (uncertainty, scale types, and derived measurement) in general conceptual framework.

## 4. Explanation of key error and uncertainty concepts and terms

**Abstract -** *In the formal expression of any measurement result the measurand value must be stated together with an estimation of its quality, that reports all the non-idealities affecting the measurement procedure with respect to both its definition and its empirical accomplishment. Traditionally accounted for in terms of errors, such a quality estimate is evaluated and formalized as a measurement uncertainty, that can be assigned by suitably combining the available objective and subjective information according to a standard formal procedure. This procedure is briefly discussed and a practical example of its application is shown.*

### 4.1. Measurement results and their quality

Measurement is a peculiar means of acquiring and formally expressing information about empirical systems. It is aimed therefore at setting up a bridge between the empirical world and the linguistic/symbolic world, the domains of the systems under measurement and measurement results respectively.

A fundamental evidence is that these two realms exhibit extremely different characteristics. Empirical systems are embedded in the space-time universe, and this generates their space and time dependency: any system is only partially isolated from its environment and its dynamics forces to distinguish between the system itself and its temporal versions, i.e., the system states. On the other hand, symbolic entities such as numbers are coextensive with their definitions (in a sense: they *are* their definitions), so that they are always identical to themselves. Paradigmatically, noise exists in the empirical realm, not in the symbolic one; real numbers exist in the symbolic realm, not in the empirical one.

Whenever the two realms interact with each other, as measurement does by means of the mediation of quantities, these diversities (1) require to introduce a concept of quality of the symbols (in our case measurement results) chosen as representatives for empirical states and (2) are the cause of several issues affecting such a quality.

The typical operative context of measurement, that can be presented as follows:

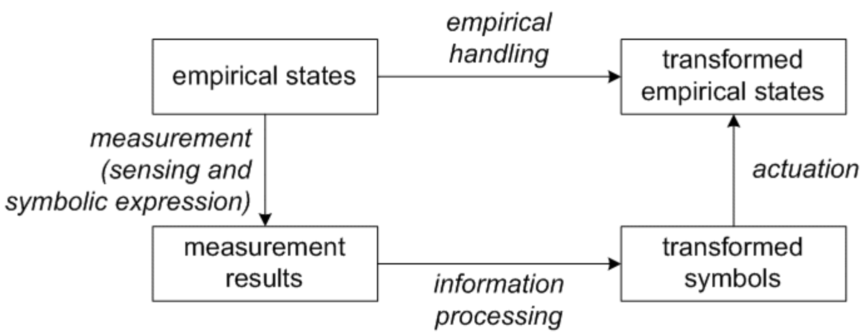


Figure 1 – The role of measurement in the relations between empirical and symbolic realms.

shows that the required empirical results ("the output") can be in principle obtained as the transformation of the same empirical states ("the input") by a direct manipulation (an "empirical procedure") or a transduction to information entities, to be processed and finally transduced back to the empirical realm (an "informational procedure"):

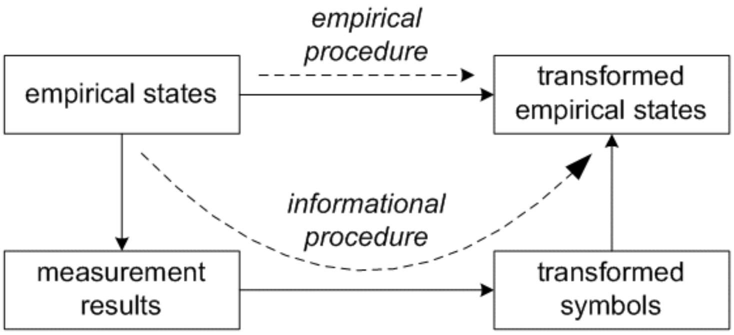


Figure 2 – The (possible) equivalence of empirical and informational procedures.

The benefits of informational procedures are commonly recognized (basically due to the fact that it is much easier to deal with symbols than with empirical things), but they depend on the faithfulness of measurement results as representative entities for the corresponding empirical states.

Such a faithfulness, and therefore the quality of measurement results, is limited in consequence of causes related to:

- the model of the system under measurement: incompleteness, if not even faults, in the definition of the measured quantity (the measurand), as in the case of an ill-characterized system dynamics or an only partial identification of the quantities influencing the measurand;
- the operative accomplishment of the measurement procedure: poor repeatability, or stability, or selectivity of the adopted measuring system, if not even faults in its usage.

The unavoidable presence of such flaws is the reason requiring us to state any measurement result by expressing in symbols a measurand value together with an estimation of its deemed quality.

## 4.2. The concept of error

It is a well-known fact that the repeatability of measurements can be increased by:

- improving the measuring system in its empirical characteristics;
- reporting the results with a reduced number of significant figures,

i.e., by adjusting the sensing device or modifying the symbolic expression respectively:

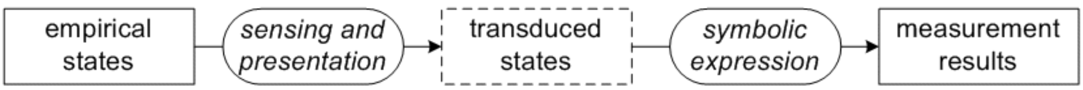


Figure 3 – Abstract schematization of a measurement.

The repeatability of a measurement, and in more general term its quality, is therefore a relative characteristic, to be evaluated in reference to the goals for which the operation is performed and the available resources (in epistemological terms this can be thought of as a confirmation that a concept of absolute, or complete, precision is simply meaningless).

It is amazing in this perspective to note that the indication of the estimated quality of the results became customary in physical measurement only in the late XIX century, and however several decades after the Theory of Errors provided by Gauss at the beginning of that century. A plausible reason of this can be recognized in the commonly (in the past) assumed hypothesis that measurable quantities are characterized by a perfectly precise “true value”. The choice to adopt the concept of error to model and formalize a less-than-ideal quality of measurements originates from this hypothesis: any discrepancy between the measuring system outputs and the measurand true value should be taken into account as an error, and correspondingly dealt with.

However:

- an error can be recognized as such only if a corresponding “right entity” exists;
- errors can be corrected only if their corresponding “right entities” are known;
- true values, that play the role of such “right entities” in the case of measurement, are in principle unknown (otherwise measurement itself would be useless…) and cannot be operatively determined.

These assertions imply that the Theory of Error is grounded on metaphysical, empirically inapplicable, bases. Consider the following two statements:

- “at the instant of the measurement the system is in a definite state”;
- “at the instant of the measurement the measurand has a definite value”.

Traditionally they would be considered as synonymous, whereas their conceptual distinction is a fundamental fact of metrology: the former represents a basic assumption for measurement (we are not considering here measurement in quantum mechanics), while the latter is epistemically unsustainable and however operationally irrelevant. Measurement results are symbolic, and not empirical, entities: what in the measurement is *determined*, and therefore considered pre-existing, is the system state, not the measurand value that is instead *assigned* on the basis of the instrument reading and the calibration information.

### 4.3. The concept of uncertainty

The search of a more adequate framework reached a crucial point about thirty years ago, when it was understood that a common approach for modeling and formally expressing a standard parameter describing the quality of measurement results was a condition to establish a strict co-operation among the national calibration laboratories. To build up and maintain a mutual confidence between accreditation bodies and compatibility for their calibration certificates required to have the quality of their measurement results evaluated and expressed according to some harmonized protocol. To this goal the *International Committee for Weights and Measures* (CIPM), started a project together with several international organizations involved in standardization (ISO, IEC, OIML, …): its final result is the *Guide to the Expression of Uncertainty in Measurement* (GUM), first published in 1993 and later introduced as a Standard by each of such organizations. While originally intended for calibration laboratories, the GUM is presently to be considered as the basis for expressing the results of any measurement performed in accordance with an international Standard.

According to the GUM, the uncertainty of a measurement result is “a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand”. Apart from this rather classical definition, the most important innovation of the GUM stands in its recognition that the uncertainty of measurement results can be evaluated according to two distinct and complementary methods:

- some uncertainties, designated as “of type A”, are computed as suitable statistics of experimental data, usually obtained as repeated instrument readings;
- some other uncertainties, designated as “of type B”, are instead estimated on the basis of the observer’s personal experience and the available a priori information, and therefore express a degree of belief on the possible measurand values.

The recognition that even measurement, an operation traditionally deemed as the paradigm of objective information acquisition, requires the introduction of some subjective evaluation is of capital importance. Therefore the shift from “error” to “uncertainty” is far more than a terminological issue, and witnesses a conceptual transition from an ontological position to an epistemic one: according to the GUM standpoint, to establish the quality of measurement results is an issue related to the *state of knowledge* of the measurer, and therefore “absolute quality” cannot be reached simply because some *intrinsic uncertainty* is always part of the measurement system.

The possible sources of uncertainty that the GUM itself lists are exemplar at this regards: together with the “variations in repeated observations of the measurand under apparently identical conditions”, the reason usually recognized for random variability, and some causes related to instrumental issues such as “approximations incorporated in the measurement procedure” and “instrument resolution or discrimination threshold”, the GUM identifies several epistemic sources, and among them the incomplete definition of the measurand and the imperfect realization of its definition.

## 4.4. Characterizing a measurement with its uncertainty

To accomplish a measurement process three distinct activities must be sequentially performed:

- *acquisition*: by means of a sensing device the measurand is transduced to a quantity suitable for direct access by the measurer (e.g., the angular position of a needle with respect to a reference scale), possibly through the mediation of an “intermediate” quantity (a typical role for electrical quantities) to drive processing and presentation devices;


Figure 4 – Abstract schematization of the empirical component of a measurement.

- *evaluation*: the access to the transduced quantity (i.e., the instrument reading) concludes the empirical part of the operation; by gathering and processing the available information (the transduced quantity itself, together with everything is known on the measurement system: the measurand definition and realization, the instrument calibration diagram, the values of relevant influence quantities, …) the measurer evaluates the measurand value and its uncertainty; this inferential process is based on both objective and subjective information;

- *expression*: the obtained information is expressed in symbolic form according to an agreed formalization.

It should be noted that the same information could be in principle expressed in different forms for different needs, by adopting, typically, a statistical or a set-theoretical formalization (or some generalization of the latter, as in the case of representations based on fuzzy sets: we will not deal with such generalizations here). Consider the traditional indication, $x \pm y$, that admits two distinct interpretations:

- the measurand value is expressed as the scalar *x*, with *y* as its estimated uncertainty;
- as the measurand value the whole interval [*x*–*y*, *x*+*y*] is taken, whose half-width, *y*, expresses the quality (sometimes called *precision* of such a measurement result.

Neither of them is the "right one": they should be selected according to the specific application requirements. The GUM adopts this approach, and while basing its procedure on the first interpretation recognizes that "in some commercial, industrial, and regulatory applications, and when health and safety are concerned", it is often necessary to express the measurement results by means of intervals of values.

Measurement results must be therefore assigned according to the goals for which the measurement is performed; they are adequate (and not "true") if they meet such goals. By suitably formalizing them, the measurer is able to express the available information of both the measurand value and its estimated quality. Quoting the GUM again, no method for evaluating the measurement uncertainty can be a "substitute for critical thinking, intellectual honesty, and professional skill": indeed "the quality and utility of the uncertainty quoted for the result of a measurement ultimately depends on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value".

## 4.5. The expression of measurement results and their uncertainty

For both type A and type B evaluation methods, the GUM assumes that measurands (but the same holds for all the quantities involved in the measurement system: influence quantities, correction factors, properties of reference materials, manufacturer or reference data, …) can be formalized as random variables, and as such characterized by statistical parameters:

- the measurand value is estimated as the mean value of the random variable; in the case of type A evaluations, for which an experimental population *X* of *n* repeated reading data $\{x_i\}$ is available, it is computed as:

$$m(X) = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{1}$$

- the uncertainty of the measurand value is estimated as the standard deviation of the measurand value, being itself a random variable; this parameter is termed by the GUM *standard uncertainty* and denoted $u(m(X))$; in the case of type A evaluations it is computed as:

$$u(m(X)) = \sqrt{\frac{1}{n}\frac{1}{n-1}\sum_{i=1}^{n}(x_i - m(X))^2} \qquad (2)$$

Measurement results can be then reported for example as $m_S$ = 100,021 47(35) g, meaning that the evaluated mass *m* of the system S (whose specification should include the indication of the operative condition in which the measurement has been performed) is 100,021 g with a standard uncertainty of 0,35 mg.

The same couple of values (measurand value, standard uncertainty) is adopted to express measurement results as intervals. To this goal a *coverage factor k* (typically in the range 2 to 3) is introduced, such that $U(X)=ku(m(X))$, termed *expanded uncertainty*, is adopted as the half-width of the interval representing the measurement result: $[m(X)-U(X), m(X)+U(X)]$, commonly written as $m(X) \pm U(X)$ (if the probability distribution of the random variable is known this interval can be thought of as a *confidence interval*, whose confidence level is depends on *k*).

In the case of *derived measurement*, i.e., when the measurand *Y* is a quantity depending on *N* input quantities $X_i$, $i=1,\ldots,N$:

$$Y = f(X_1,\ldots,X_N) \qquad (3)$$

and for each quantity $X_i$, the estimated value $m(X_i)$ and uncertainty $u(m(X_i))$ are given, the issue arises of how to obtain the corresponding values $m(Y)$ and $u(m(Y))$ for Y.

The measurand value $m(Y)$ is simply obtained by introducing the estimates $m(X_i)$ in the model function *f*:

$$m(Y) = f(m(X_1),\ldots, m(X_N)) \qquad (4)$$

The uncertainty $u(m(Y))$ is instead evaluated by means of the so-called *law of propagation of uncertainty*, that for statistically non-correlated quantities is:

$$u^2(m(Y)) = \sum_{i=1}^{N} c_i^2 u^2(m(X_i)) \qquad (5)$$

where the “sensitivity coefficients” $c_i$ that define the extent to which *Y* is influenced by variations of the input quantities $X_i$ are computed as:

$$c_i = \frac{\partial f}{\partial X_i} \text{ evaluated at } X_i = m(X_i)$$

In the general case of correlated input quantities (i.e., their covariance $u(m(X_i), m(X_j)) \neq 0$), the equation (5) becomes:

$$u^2(m(Y)) = \sum_{i=1}^{N} c_i^2 u^2(m(X_i)) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_i c_j u(m(X_i), m(X_j)) \qquad (6)$$

in which the combined standard uncertainty of the measurement result $m(Y)$ is computed on the basis of a first-order Taylor series approximation of equation (3).

### 4.6. The procedure for assigning the measurement uncertainty: an example

Measurement uncertainty is a *pragmatic* parameter: its value is not intrinsic to the measurand but is to be established in reference to the specific goals according to which the measurement is performed. No “true uncertainty” exists, and the preliminary step of a procedure aimed at assigning a value to the uncertainty of a measurand value is therefore to decide a *target* *uncertainty*, the maximum value of uncertainty compatible with the given goals. In any step of the procedure, if the estimated value is reliably considered less than such a target uncertainty, then the procedure should be stopped with a positive result: the measurand can be evaluated with a satisfying uncertainty, and no further resources are required to refine the procedure. On the other hand, whenever the estimated uncertainty becomes greater than the target uncertainty the procedure must be definitely stopped with a negative outcome, conveying the information that better measurements are required to meet the specified target uncertainty.

According to the GUM viewpoint, *any* measurand $Y$ should be actually evaluated by derived measurement, i.e., by firstly identifying its dependence on a set of "input quantities" $X_i$, such as influence quantities, calibration parameters, correction factors, … For each $X_i$, the values $m(X_i)$ should be obtained by statistical or other methods (e.g., as part of instrument specifications), and for each of such $m(X_i)$ the corresponding standard uncertainties $u(m(X_i))$ and covariances $u(m(X_i), m(X_j))$ should be evaluated, again by either type A or type B procedures.

In the case the functional relation $f$ is known in its analytical form, the sensitivity coefficients $c_i$ can be then computed; if, on the other hand, the complexity of the measurement system prevents the explicit formalization of the equation (3), each coefficient $c_j$ can be experimentally estimated by a suitable setup of the system in which $Y$ is repeatedly measured while all the quantities $X_i$ but $X_j$ are kept constant.

When at least some $u(m(X_i))$, $u(m(X_i), m(X_j))$, and $c_i$ are available, the equation (6) can be computed to obtain an estimation of the measurand uncertainty $u(m(Y))$, to be compared to the specified target uncertainty.

## 5. Errors in digital signal systems

**Abstract -** *Characteristic of the concept of digital coding is the hypothesis that physical signals are just carriers for symbols, so that physical transformations of signals are actually dealt with as data processing operations. Correspondingly, measurement systems that process digital signals are metrologically characterized by identifying the main sources of uncertainty / error in reference to the data acquisition (usually including an analog-to-digital converter) and the data processing (also optionally performing data transmission) subsystems.*

### 5.1. Uncertainty and error sources in digital signals

Digital systems are adopted today in a broad range of measurement applications. While supported by the current remarkable innovations in microelectronics and the related technologies, the reason of the widespread usage of digital systems in measurement is grounded on the purpose itself of the operation: to extract and formally express information from the physical signals obtained by the systems under measurement.

Traditional measuring systems behave as transducers of measurands to quantities directly perceivable by human beings (such as angular deflections of needles on graduated scales), and as such their operation can be integrally described in terms of physical transformations, the interpretation of physical states as information entities being left to observers. In this case, any further data processing (leading to the so-called indirect, or derived, measurement) is accomplished by devices external to the measuring system, if not manually by the observers themselves.

On the other hand, characteristic of the very concept of digital coding is the hypothesis that physical signals are just carriers for univocally recognizable and mutually distinguishable *symbols*, so that physical transformations of signals are actually modeled and dealt with as *data processing operations* (i.e., mathematical functions) *among symbols*.

The opposition hardware-software is paradigm of this transition: while analog systems *consist of* their hardware, in the case of digital systems a progressive virtualization of the hardware layer is obtained, from *hard-wired logic systems*, to *microprocessor-based programmable systems*, to the so-called *virtual instruments*, whose operation could be even interpreted *as if* their characterizing software layer is executed on an ideal hardware subsystem.

Correspondingly to each of these *levels of abstraction*, different issues arise in the metrological characterization of the systems, and in particular in the identification of the typical sources of uncertainty / error, related to both the hardware and the (multiple) software layers. The combined uncertainty $u_Y$ summarizing the contributions of such multiple sources $u_{X_i}$ depends additively on them, as formalized by the law of propagation of uncertainty, as recommended by the ISO *Guide to the expression of uncertainty in measurement* (GUM) (the simplified version of such a law is shown here, applicable in the case of statistically uncorrelated sources):

$$u_Y^2 = \sum_{i=1}^{N} \left( \frac{\partial f}{\partial X_i} \right)^2 u_{X_i}^2 \qquad (1)$$

where $f$ is the function modeling the relation that links the measurand to its influence quantities. The equation (1) is obtained as a first-order Taylor series approximation of the model function computed in a (*N*-dimensional) point assumed “sufficiently closer” to the average values of the quantities $X_i$ and under the hypothesis that $f$ is “sufficiently linear” in the neighborhood of such a point. While usually reasonably correct in the case of instrumentation dealing with smoothly varying quantities, these assumptions could be critical for digital systems, in which non-linearities (that sometimes are very strong, such as those manifesting as the consequence of bugs in the software) are common.

Given the fundamental requirement to formalize any measurement result by expressing both a measurand value and an estimation of its uncertainty, the usage of digital signals and systems (particularly if with software control) usually implies to trade off flexibility with complexity.

## 5.2. List of typical digital signal uncertainties and errors

While in some specific cases digital systems integrally operate on digitally coded entities (e.g., in some cases of counting, in which the measurand is inherently discrete), they are widely used also in measurement of continuously varying quantities so that a preliminary stage of analog-to-digital conversion is implied. Once such a transduction has been completed the operations are performed on coded symbols, i.e., on a purely algorithmic basis. The results are then fed into a device acting as output transducer which is sometimes required to convert the digital symbols back to analog signals.

As a consequence, a metrological characterization of digital systems involves the analysis of their behavior in reference to such three general components, each of them being affected by specific causes of uncertainties / errors.

* *The input subsystem* is aimed at acquiring information on the measurand from the environment and, when needed, converting it in digital form. Its general structure includes then a sensor, a signal conditioning component, and an analog-to-digital converter (ADC, that in PC-based systems is usually part of a data acquisition card). Digital signals are obtained as the

output of such a subsystem; hence, strictly speaking the input subsystem does not contribute to the budget of system uncertainties / errors related to digital signals. On the other hand, ADC characteristics and behavior significantly influence the quality of the generated digital signals (conceptually definable as the degree of correspondence with the originating analog signals and operatively affecting the possibility to reconstruct them from the converted digital signals).

* *The data processing subsystem* is aimed at dealing with digitally coded entities to transform them by means of suitably implemented algorithms and / or to transfer them to remote devices. Uncertainties / errors can appear in both hardware and software layers, because of the presence of physical factors modifying the quantity on which the symbols are coded and low quality of algorithms (or their implementations) adopted in the processing of such symbols respectively. The latter issue grows in relevance as the software adopted for metrological purposes becomes more and more complex, as is the case of spreadsheets or virtual instruments. The current developments in this area are particularly important, as witnessed by the emerging applications of pattern recognition, automatic control, and data fusion based on the so-called soft computing paradigm, in which techniques such as neural networks and fuzzy logic inference are used to exploit uncertainty and partial information.

* *The output subsystem* is finally aimed at making the processed data available to users and user devices (e.g., actuators of control systems), while possibly converting such data to a corresponding analog form. At this stage raw data produced by the measuring system must be converted to information meaningful to the intended users and useful to them. The sources of possible uncertainties / errors in the expression of measurement results from the digital signals representing the instrument readings are multiple, all basically *related to the mathematical model of the measurement system* (we will not deal with this topic here). While traditionally assigned to human beings, the definition and the metrological qualification of this model is the main task of the knowledge-based intelligent instruments.

## 5.3. Digital signal uncertainties and errors in data acquisition

The digitalization of analog signals usually implies their time and amplitude discretization, the two basic parameters qualifying such operations being the sampling rate and the amplitude resolution (also called bit depth) of quantization, measured in samples per second and bits respectively. Even in the case of an “ideal” behavior of the ADC, the limitations in size of the data storage devices and in bandwidth of the data transmission channels are sources of errors on the generated digital signals:

- the sampling theorem assures that the information conveyed by an analog signal is integrally maintained whenever the signal is sampled at a rate greater than twice its bandwidth (for most applications the time interval between samples is kept constant); the usual technique of low-pass (sometimes band-pass) anti-aliasing filtering is in fact a trade-off between two systematic errors: its application allows to avoid aliasing effects but removes any information contained in the cut-off portion of the signal spectrum;
- the number of intervals (sometimes called channels or cells) in which the amplitude range is subdivided in quantization specifies the quantizer resolution, i.e., the length of the binary word coding each sample, and thus establishes the amount of the error introduced by the quantization; in the simplest case of uniform quantization, when all the intervals have the same half-width $a$, each sample of amplitude $x$ is associated with a channel $i$ whose mid

point (dealt with as the reference value to be coded) is $c_i$ : the quantization error is then $x - c_i$ , corresponding to a maximum quantization error of $\pm 0.5$ least significant bits (LSBs) and a null average quantization error; here again a trade-off is implied: to reduce the quantization error the bit depth of the code word must be increased (in other terms, to enhance the ADC accuracy its precision must be also increased).

To characterize the actual behavior of a physical ADC some further parameters have to be taken into account, such as (internal and external) noise, settling time, short-term and long-term stability (the former is sometimes called repeatability), offset, linearity of gain, and (in the case two or more signals are acquired at the same time) cross-talk. It is usual that the specifications for such parameters are directly given by the ADC manufacturer as the interval $\pm a$ that surely (i.e., with probability =1) contains the corresponding values / errors. This is the typical case in which the ISO GUM recommends type B uncertainty evaluations based on uniform probability distributions: the corresponding standard uncertainties are then computed as $a/\sqrt{3}$ and combined by means of equation (1).

## 5.4: Digital signal uncertainties and errors in data processing

The simplest kind of data processing is the one performed by systems computing the identity function, i.e., producing as their output the same symbols given at their input, as typically behaves an ideal digital transmission channel. In this case the presence of errors (generally caused by noise sources external to the channel) is modeled in statistical terms, by recognizing that for each input symbol $x_i$ the channel does not deterministically produce an output symbol $y_j$ but a conditional probability distribution $P(y_j|x_i)$ (for binary channels $x_i, y_j \in \{0,1\}$, and $P(0|x_i) + P(1|x_i) = 1$ ). The average value of $-\log_2(P(x_i|y_j))$ , called *equivocation* and computed from $P(y_j|x_i)$ by means of the Bayes theorem, represents the average information lost in the transmission process because of errors. From the channel equivocation $H(X|Y)$ and the source entropy $H(X)$ the channel capacity $C$ is computed:

$$C = \max_X (H(X) - H(X|Y)) \qquad (2)$$

a basic informational quantity, measured in bit/symbol (and more usually in bit/s by multiplying it by the rate of symbol transmission over the channel), whose physical grounds are clearly identified in the fundamental relation:

$$C = W \log_2(1 + S/N) \qquad (3)$$

where $W$ and $S/N$ are the channel bandwidth and signal-to-noise ratio respectively.

In the case the information flowing from the source has a rate lower than the capacity $C$ of the channel, several techniques can be adopted to reduce the probability of error at the receiver, all based on the introduction of redundancies and aimed at either error recognition or correction.

Typical applications of digital signal processing in measurement are digital filtering and DTF / FFT computation, but also higher-level operations are now common, e.g., to compute statistical parameters as in the case of DC / RMS measurement. The fundamental parameters qualifying the arithmetic of a processor are its overflow, underflow, and roundoff error thresholds.

In the common case of the floating-point number representation (in which numbers are expressed as $(-1)^a \cdot b \cdot 10^c$ where $a \in \{0,1\}$, the mantissa $b \in [1,10)$ has a fixed number of digits, and the exponent *c* is an integer spanning among two fixed values), the overflow and the underflow thresholds depend on the maximum positive and negative values of the exponent respectively. On the other hand, roundoff errors depend on the number of digits reserved for the mantissa, and are expressed in terms of the *machine precision,* a value generally related to the characteristics of not only the processor arithmetic-logic unit (ALU) but also the adopted software platform or compiler: this is an important source of complexity in the metrological qualification of data processing modules (a commonly implemented reference for the values of these parameters is the IEEE Standard, see Table 1).

| Machine parameter | Single Precision (32 bits) | Double Precision (64 bits) |
|---|---|---|
| Machine precision | $2^{-24} \approx 5{,}96 \cdot 10^{-8}$ | $2^{-53} \approx 1{,}11 \cdot 10^{-16}$ |
| Underflow threshold | $2^{-126} \approx 1{,}18 \cdot 10^{-38}$ | $2^{-1022} \approx 2{,}23 \cdot 10^{-308}$ |
| Overflow threshold | $2^{128}(1-\varepsilon) \approx 3{,}40 \cdot 10^{38}$ | $2^{1024}(1-\varepsilon) \approx 1{,}79 \cdot 10^{308}$ |

Table 1 - Values of machine parameters in IEEE floating point arithmetic

The data processing subsystem is usually so complex that instead of identifying all the relevant sources of uncertainty, as it would be required to apply equation (1), a black box solution is sometimes adopted for its metrological qualification: a *reference data set* is chosen, containing a collection of sampled input data with the corresponding expected output, such input data are fed into the subsystem, and the results are compared with the references. From the analysis of the obtained error an estimation of the uncertainty of the data processing results is then inferred.

Finally, the contribution of the possible hardware faults (and correspondingly the degree of fault tolerance of the system) should be taken into account for a complete metrological qualification of the system.

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